Appendix C

Proof of Performance Equation

This appendix gives a proof that the overall performance equation must take the form of a product of independent functions, where each function is a function of exactly one workload parameter (see Figure 5.1 on page 83).

I first show this is true with two workload parameters A and B.

Given: Performance equation $f(A,B)$

and $\dfrac{f(A,B_1)}{f(A,B_2)} = constant$

Show: $f(A,B) = g(A) \times h(B)$

Proof:

$$\frac{f(A,B)}{f(A,B_1)} = constant \text{ (for all A with B fixed)}$$

$$\frac{f(A,B)}{f(A,B_1)} = \frac{f(A_1,B)}{f(A_1,B_1)}$$

$$f(A,B) = \frac{f(A_1,B)}{f(A_1,B_1)} \times f(A,B_1)$$

$$f(A,B) = g(B) \times h(A)$$

where $g(B) = \dfrac{f(A_1,B)}{f(A_1,B_1)}$

and $h(A) = f(A,B_1)$

Q.E.D.

Next, I generalize this to multiple workload parameters A, B, C, D...

$$\frac{f(A,B,C,D,\cdots)}{f(A,B_1,C,D,\cdots)} = \frac{f(A_1,B,C,D,\cdots)}{f(A_1,B_1,C,D,\cdots)}$$

$$f(A,B,C,D,\cdots) = \frac{f(A_1,B,C,D,\cdots)}{f(A_1,B_1,C,D,\cdots)} \times f(A,B_1,C,D,\cdots)$$

This equation has the form of a product of functions, with each each term having at least one less independently-varying parameter. By recursing down to two workload parameters and using the commutative property of multiplication/division, the overall performance equation will reduce to a product of functions, with each function being a function of one workload parameter.

Q.E.D.