FINITE-TIME IMPLICATIONS OF RELAXATION TIMES FOR STOCHASTICALLY MONOTONE PROCESSES.

by

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Abstract

For a continuous-time finite state Markov process with stationary distribution π, it is well-known that $P_i(X_t = j) - \pi_j$ is $O(e^{-\lambda t})$ as $t \to \infty$, for a certain λ. For a stochastically monotone process for which the reversed process is also stochastically monotone, one can obtain bounds valid for all t. Precisely, $\sum_i \pi_i \max_j |P_i(X_t \le j) - \pi[0,j]| \le 2(\lambda t + 2)\exp(-\lambda t)$. The proof exploits duality for stochastically monotone processes.

1. Introduction. The technical result stated in the abstract is a small contribution to a much broader project, which we now describe. Under many circumstances, a Markov process X_t which converges to a stationary distribution π does so asymptotically at an exponential rate:

$$P(X_t \in A) - \pi(A) \sim c_A \exp(-\lambda t) \quad \text{as } t \to \infty; \tag{1.1}$$

where the rate constant λ has an eigenvalue interpretation. In applications to the natural sciences, $\tau \equiv 1/\lambda$ is called the relaxation time of the process. The relaxation time is used to indicate the order of magnitude of quantities such as the time for the effects of an external shock to a system to wear off, or the time for sample averages of a functional of the process to approach the limiting ergodic average. See e.g. Blanc and van Doorn (1985), Gardiner (1983), Haken (1978).

It is realized that these informal interpretations may be misleading because, for instance, the time for the asymptotic relation (1.1) to come into play may be larger that the time interval under study. Thus a natural theoretical question is:

(1.2) what can be said rigorously about the finite-time behavior of a Markov process, given its stationary distribution π and its relaxation time τ?

One aspect of this question, perhaps not the most interesting in applications but easy to formalize, is the following. Consider the stationary process $(X_t; t \geq 0)$, and let $d(t)$ be some measure of dependence between X_t and X_0. Typically $d(t) \sim c\exp(-\lambda t)$ as $t \to \infty$; when can we get a bound on $d(t)$ valid for all $t \geq 0$ which involves the process only through the relaxation time?

We shall study continuous-time finite state space Markov processes. Informally, for understanding essentially finite-time properties there is no loss of generality in restricting attention to finite state spaces, since a general state space process on a finite time interval can be arbitrarily well approximated by a finite state process. Write $(X_t;\, t \geq 0)$ for the process, $I = \{i,j,k,...\}$ for the state space, $Q = (q_{i,j})$ for the matrix of transition rates, with $q_{i,i} = -\sum_{j \neq i} q_{i,j}$, and assume the process is irreducible so there exists a unique stationary distribution $\pi = (\pi_i)$. Then, in vector-matrix notation,

$$\pi Q = 0.$$

The eigenvalue equation

$$xQ = -\lambda x$$

has solutions $0 = \lambda_1, \lambda_2, \cdots, \lambda_{|I|}$ which can be ordered so that $0 < \mathrm{Re}(\lambda_2) \leq \mathrm{Re}(\lambda_3) \leq .....$ Write

$$\lambda = \mathrm{Re}(\lambda_2) \tag{1.3}$$

and define the relaxation time to be $\tau = 1/\lambda$.

For our measure of dependence d(t) there are a bewildering variety of "mixing coefficients" or "coefficients of ergodicity" to choose from: see e.g. Seneta (1979), Bradley (1983). One variant is

$$(1.4) \qquad d_2(t) = \sup_{f,g} \text{correlation}(f(X_0), g(X_t))$$

where X is taken to be the stationary process. For a reversible process X, that is to say when

$$\pi_i q_{i,j} = \pi_j q_{j,i} \quad \text{for all } i,j$$

standard spectral theory (i.e. matrix diagonalization) shows that the eigenvalues are all real and that

$$(1.5) \qquad d_2(t) = \exp(-\lambda t);\ t \geq 0$$

for λ as at (1.3). This seems to be the only known result giving bounds for some d(t) in terms of λ; and it is natural to ask whether analogous results hold under conditions other than reversibility.

We consider the case where the state space $I = \{0,1, \cdots, N-1\}$. For distributions $x = (x_i)$, $y = (y_i)$ on I, x is stochastically smaller than y ($x \underset{st}{\leq} y$) iff $F_x(i) \geq F_y(i)$ for all i, where F denotes the distribution function. The process is <u>stochastically</u> <u>monotone</u> if

$$P_i(X_t \in \cdot\,) \underset{st}{\leq} P_j(X_t \in \cdot\,);\ \text{all } i < j,\ t \geq 0.$$

In terms of Q, this condition becomes: for all i,

$$\sum_{k \geq j} q_{i,k} \leq \sum_{k \geq j} q_{i+1,k};\ \text{all } j > i+1$$

$$\sum_{k \leq j} q_{i,k} \geq \sum_{k \leq j} q_{i+1,k};\ \text{all } j < i.$$

Informally, the class of stochastically monotone processes is the class of processes which respect the order structure of the line. A natural distance between distributions x, y on I which involves the order structure is

$$\Delta(x,y) = \max_i |F_x(i) - F_y(i)|.$$

Then a measure of dependence between X_0 and X_t in the stationary process can be defined by

$$\begin{aligned} d_1(t) &= E\Delta(P(X_t \in \cdot\,|X_0), P(X_t \in \cdot\,)) \\ (1.6) \qquad &= \sum_i \pi_i \Delta(P_i(X_t \in \cdot\,),\pi) \\ &= \sum_i \pi_i \max_j |P_i(X_t \leq j) - \pi[0,j]|. \end{aligned}$$

(1.7) Theorem. *If the process* (X_t) *and the time-reversed process* $(\hat{X}_t)$ *are both stochastically monotone, then*

$$d_1(t) \le 2(\lambda t + 2)\exp(-\lambda t); \text{ all } t \ge 0.$$

Recall that the time-reversed process $(\hat{X}_t)$ has transition rates

$$\hat{q}_{i,j} = \pi_j q_{j,i}/\pi_i. \tag{1.8}$$

An example will be given in Section 4 to show that no such result holds if only (X_t) is assumed stochastically monotone. Such examples seem rather surprising, and suggest that the class of chains for which we can justify finite-time inferences from asymptotic relaxation times is rather limited.

The proof relies on qualitative properties of leading eigenvectors. These properties are derived via duality in Section 2, and applied to the proof of Theorem 1.7 in Section 3.

Warning: ambiguous terminology. Our notion of "stochastically monotone" is the standard one of monotonicity "in space", and must not be confused with the notion of "stochastically monotone in time" used in van Doorn (1981). Our notion (2.1) of "dual process" is a kind of duality "in space" and must not be confused with the time-reversed process (in general process theory, "dual process" is used as a synonym for "time-reversed process"). Finally, we use "increasing" in the weak sense, to mean "non-decreasing".

2. Duality and eigenvectors. Seigmund (1976) proved the following result; see also Clifford and Sudbury (1985) for an elegant proof, and Liggett (1985) Section 2.3 for related developments. These references concentrate on the infinite-state case, but the finite-state case is similar.

(2.1) Proposition. *Let* (X_t) *be a stochastically monotone Markov process on states* $\{0,1, \cdots, N-1\}$. *Then there exists a Markov process* (Y_t) *on states* $\{0,1, \cdots, N\}$ *which is dual in the sense*

$$P_i(X_t \geq j) = P_j(Y_t \leq i) \textit{ for all } 0 \leq i \leq N-1, \ \ 0 \leq j \leq N, \ \ t \geq 0.$$

Note that this duality relation implies

(2.2) 0 and N are absorbing states for Y.

We shall show later (2.12) that there is no loss of generality in supposing

(2.3) $\{1,2, \cdots, N-1\}$ forms a communicating class for Y.

We can now appeal to the continuous-time analogue of Perron-Frobenius matrix theory (Seneta (1981)) to obtain the following result.

(2.4) Proposition. *For a Markov process* (Y_t) *on states* $\{0, \cdots, N\}$ *satisfying (2.2) and (2.3), let* A *be the matrix of transition rates, restricted to* $\{1,2, \cdots, N-1\}$. *The eigenvalue equations*

$$\alpha A = -\lambda\alpha; \ \ A\beta = -\lambda\beta$$

have a solution λ *which is real, strictly positive, and strictly less than the real part of any other eigenvalue. The corresponding eigenvectors* α, β *can be normalized so that* α *and* β *are strictly positive,* $\Sigma\alpha_i = 1$, *and* $\Sigma\alpha_i\beta_i = 1$. *Then*

(2.5) $$P_i(Y_t = j) \sim \beta_i\alpha_j\exp(-\lambda t) \text{ as } t \to \infty; \ 1 \leq i,j \leq N-1.$$

These two Propositions are the main ingredients of our proof. For the rest of this section we record some immediate consequences. Consider (X_t), (Y_t) as in Proposition 2.1. The matrix A of transition rates of Y is related to the matrix Q for X by:

(2.6) $$\begin{aligned} a_{i,j} &= \sum_{k<i} q_{j-1,k} - \sum_{k<i} q_{j,k} \quad (i < j) \\ &= \sum_{k\geq i} q_{j,k} - \sum_{k\geq i} q_{j-1,k} \quad (i > j), \end{aligned}$$

with the convention that $q_{i,j} = 0$ outside $0,1, \cdots, N-1$: see Seigmund (1976). Given eigenvectors u, v of A associated with an eigenvalue λ

$$uA = -\lambda u; \ Av = -\lambda v,$$

write $u_0 = v_0 = u_N = v_N = 0$. It can be checked that $xQ = -\lambda x$, $Qy = -\lambda y$ for

$$y_j = \sum_{i \le j} u_i;\ x_i = u_i - u_{i+1}.$$

Thus we have

(2.7) Corollary. *The non-zero eigenvalues of* Q *coincide with the eigenvalues of* A. *In particular, the dominant eigenvalue* $\lambda = \lambda_2$ *of* Q *defined at* (1.3) *is real.*

This fact that λ_2 is real for a stochastically monotone Markov process does not seem obvious (to the author) without the duality argument (for a general finite Markov process, λ_2 need not be real). Henceforth write λ for λ_2. Let α,β be the eigenvectors of Proposition 2.4, and let

(2.8) $$\gamma_j = \beta_j - \beta_{j+1}$$
$$\delta_j = \sum_{i \le j} \alpha_i - c, \text{ for } c = \sum_{i \le j}\sum \alpha_i \pi_j.$$

It can be checked, using (2.6), that

(2.9) $$\gamma Q = -\lambda\gamma;\ Q\delta = -\lambda\delta.$$

Then (2.5) and the duality relation give

(2.10) Corollary. $P_i(X_t = j) - \pi_j \sim \delta_i\gamma_j \exp(-\lambda t)$ as $t \to \infty$; $0 \le i,j \le N-1$.

Next, stochastic monotonicity yields some qualitative properties of these eigenvectors.

(2.11) Lemma. (a) δ_i *is increasing in* i.

(b) $\sum_{j \le k} \gamma_j \le 0$ *for all* k.

(c) If the time-reversed process $\hat{X}_t$ *is also stochastically monotone then* γ_i/π_i *is increasing in* i.

Proof. (a) is clear from (2.8), since α is a probability distribution. Note also that δ is non-constant. Fix k. Stochastic monotonicity implies $P_i(X_t \le k)$ is decreasing in i. So Corollary 2.10 implies $\delta_i \sum_{j \le k} \gamma_j$ is decreasing in i. In view of (a) this means $\sum_{j \le k} \gamma_j \le 0$, giving (b). Next, the time-reversed process $\hat{X}_t$ satisfies $P_i(\hat{X}_t = j) = \pi_j P_j(X_t = i)/\pi_i$. So by Corollary 2.10

$$P_i(\hat{X}_t = j) - \pi_j \sim \hat{\delta}_i\hat{\gamma}_j \exp(-\lambda t) \text{ as } t \to \infty; \text{ where}$$
$$\hat{\gamma}_i = \delta_i\pi_i,\ \hat{\delta}_i = \gamma_i/\pi_i.$$

If $\hat{X}$ is stochastically monotone, then (a) implies $\hat{\delta}_i$ is increasing, and this gives (c).

<u>Remark.</u> γ is normalized so that $\Sigma\gamma_i = 0$, so (c) is a stronger property than (b).

(2.12) <u>Remark.</u> The irreducibility condition (2.3) ensures that λ in (2.4) has multiplicity 1. Given (X_t) satisfying the hypotheses of Theorem 1.7 with matrix Q and for $\epsilon > 0$, we can construct a transition rate matrix Q^ϵ such that

(a) $|q_{i,j}^\epsilon - q_{i,j}| \leq \epsilon$ for all i,j;

(b) Q^ϵ has the same stationary distribution π as Q does;

(c) the associated chain (X_t) satisfies the hypotheses of Theorem 1.7;

(d) the dual process (Y_t) satisfies the irreducibility condition (2.3).

Since $\lambda^\epsilon \to \lambda$ as $\epsilon \to 0$, the truth of Theorem 1.7 under condition (2.3) will imply its truth without that condition.

3. Proof of Theorem 1.7. Let (Y_t) be the dual process to (X_t), let β be the eigenvector associated with λ in (2.4), extended to $\beta_0 = \beta_N = 0$. Write $F(i) = \sum_{j=0}^{i-1} \pi_j$. The key facts are

(3.1) $F(Y_t)$ is a martingale;

(3.2) $e^{\lambda t}\beta_{Y_t}$ is a martingale;

(3.3) $(\beta_{i+1} - \beta_i)/\pi_i$ is decreasing in i.

Indeed, (3.3) follows from (2.11) (c) and (2.8); and (3.2) is a standard consequence of the eigenvalue equation $A\beta = -\lambda\beta$. And the duality relation implies $P_j(Y_j \geq i-1) = P_i(X_t \leq j-1) \to F(j)$ as $t \to \infty$, and so $F(j) = P_j(Y_\infty = N)$, giving (3.1).

Here is a simple lemma, which we shall prove later.

(3.4) Lemma. *For* $0 \leq z \leq 1$ *let* $z^* = \min(z, 1-z)$. *Then for a random variable* $0 \leq Z \leq 1$,

$$|P(Z \geq z) - EZ| \leq (1 + 1/z^*)EZ^*.$$

(3.5) Proposition.

$$|P_i(Y_t > j) - F(i)| \leq (1 + 1/G(j))\exp(-\lambda t);\ \ 0 \leq i,j \leq N-1,\ \ t \geq 0,$$

where $G(j) = \min(F(j+1), 1 - F(j), \frac{1}{2})$.

Proof. Fix i_0 such that $\beta_{i_0} = \max \beta_i$. Consider the graph through vertices $(F(i),\beta_i)$, $0 \leq i \leq N$, where the endpoints are (0,0) and (1,0), and piecewise linear between vertices. Property (3.3) says the graph is concave. Hence

$$\begin{aligned} \beta_i/\beta_{i_0} &\geq F(i)/F(i_0),\ \ i \leq i_0 \\ &\geq (1 - F(i))/(1 - F(i_0)),\ \ i \geq i_0, \end{aligned}$$

and so in particular

$$\beta_i/\beta_{i_0} \geq F^*(i)$$

in the notation of Lemma 3.4. This implies

$$\begin{aligned} E_iF^*(Y_t) &\leq E_i\beta_{Y_t}/\beta_{i_0} \\ &= e^{-\lambda t}\beta_i/\beta_{i_0} \text{ by (3.2)} \\ &\leq e^{-\lambda t}. \end{aligned}$$

Now by (3.1), $E_iF(Y_t) = F(i)$. So we can apply Lemma 3.4 and conclude that

$$|P_i(F(Y_t) \geq z) - F(i)| \leq (1 + 1/z^*)e^{-\lambda t}$$

for all $0 < z < 1$. So in particular for $F(j) < z < F(j+1)$,

$$|P_i(Y_t > j) - F(i)| \leq (1 + 1/z^*)e^{-\lambda t}.$$

The Proposition follows upon noting that

$$\sup_{F(j)<z<F(j+1)} z^* = \min(F(j+1), 1 - F(j), \frac{1}{2}).$$

To prove Theorem 1.7, note that the duality relation says $P_i(Y_t > j) = P_j(X_t < i)$. So Proposition 3.5 says, in the notation of (1.6), that

$$\Delta(P_j(X_t \in \cdot\,),\pi) \leq (1 + 1/G(j))e^{-\lambda t}.$$

And recalling that $\Delta \leq 1$ always,

$$d_1(t) \leq \sum_{j=0}^{N-1} \pi_j \max(1, (1 + 1/G(j))e^{-\lambda t}).$$

Let V have distribution π. It is easy to check that $G(V)$ is stochastically larger than a random variable U uniform on $(0,\frac{1}{2}]$. So

$$d_1(t) \leq E\max(1,(1 + 1/U)e^{-\lambda t})$$

$$\leq 2\int_0^{\frac{1}{2}} \max(1, (1 + 1/u)e^{-\lambda t})du,$$

and routine calculus yields the bound stated in Theorem 1.7.

Proof of Lemma 3.4. $Z^* = |Z - W|$, where $W = 1_{(Z \geq \frac{1}{2})}$. Consider $z \geq \frac{1}{2}$. Then

$$0 \leq P(W = 1) - P(Z \geq z)$$

$$\leq P(W - Z \geq 1 - z)$$

$$(3.6) \qquad \leq (1 - z)^{-1}E|W - Z| = EZ^*/z^*.$$

And $|P(W = 1) - EZ| = |EW - EZ|$

$$(3.7) \qquad \leq E\,|W - Z| = EZ^*.$$

Putting together (3.6) and (3.7) gives

$$(3.8) \qquad |P(Z \geq z) - EZ| \leq (1 + 1/z^*)EZ^*;\ z \geq \frac{1}{2}.$$

To get the desired result for $z < \frac{1}{2}$, observe that (3.8) is equivalent to

$$|P(Z < z) - (1 - EZ)| \leq (1 + 1/z^*)EZ^*;\ z \geq \frac{1}{2}.$$

Applying this to $1 - Z$ and $1 - z$ gives (3.8) for $z < \frac{1}{2}$.

4. A counter-example. Here we sketch an example which illustrates some limitations on the generality in which relaxation times give information about the finite-time behavior of a process.

Given N, consider the process X_t with states $\{0,1,\cdots,N-1\}$ and transition rate matrix

$$q_{i,i+1} = (N-i-1)/(N-i);\ 0 \le i \le N-2$$

$$q_{i,j} = 1/[(N-j)(N-j+1)];\ 1 \le j < i \le N-1.$$

$$q_{i,0} = 1/N;\ 1 \le i \le N-1.$$

This example does not look pathological. It is stochastically monotone and upward skip-free, both of which are familiar regularity properties. The stationary distribution π is uniform. However, it turns out that the relaxation time $\tau = 1$ (for every N). Using the skip-free property, for the stationary processes we find $P(X_0 \le \frac{1}{4}N, X_{\frac{1}{4}N} \ge \frac{3}{4}N) \to 0$ as $N \to \infty$. It is now clear that for any class **C** of processes for which a theorem of the type

> "there exists a measure of dependence d(t) and a function
> $f(t) \to 0$ as $t \to \infty$ such that for every (X_t) in **C**,
> $d(t) \le f(t/\tau)$ for all $t \ge 0$
> where τ is the relaxation time of X"

is true, the class **C** cannot contain these examples. In particular, the hypothesis of Theorem 1.7 cannot be weakened to assume only that (X_t) is stochastically monotone.

The behavior of the process X_t becomes clearer when one considers the dual process Y_t, which has states $\{0,1,\cdots,N\}$ and transition rate matrix A given by

$$a_{i,N} = 1/(N-i+1)$$

$$a_{i,i-1} = 1 - 1/(N-i+1).$$

So Y holds at i for an exponential, mean 1, time and then jumps to $i-1$ or N with probabilities chosen to make Y a martingale. It is intuitively clear that for large t

(a) given that Y_t has not been absorbed into $\{0,N\}$, it is likely to be at state 1;

(b) the chance that Y_t has not been absorbed by time t is largest for initial state $N-1$.

Analytically, one finds that the eigenvectors α,β and eigenvalue λ of Proposition 2.4 are

$$\alpha = \epsilon_1 \quad \beta = \epsilon_{N-1} \quad \lambda = 1$$

where ϵ_k is the unit vector $\epsilon_k(i) = 1_{(i=k)}$. Thus via (2.7) the eigenvectors γ,δ and eigenvalue λ controlling the asymptotic rate of convergence (2.9) of X_t to π are given by

$$\gamma = \epsilon_{N-1} - \epsilon_{N-2}; \quad \delta_0 = 1/N - 1, \ \ \delta_i = 1/N \text{ for } i \geq 1; \lambda = 1.$$

5. Miscellaneous Remarks.

(a). For infinite state space processes, the rate of convergence to the stationary distribution may or may not be asymptotically exponential, depending on whether or not 0 is an isolated eigenvalue. For discussions in various settings see Nummelin and Tuominen (1982), Sullivan (1984), Van Doorn (1985).

It is intuitively clear that Theorem 1.7 extends unchanged to the real-valued setting, provided τ is finite.

(b). It would be interesting to find a d-dimensional version of Theorem 1.7. Our technique cannot work, since in $d \geq 2$ dimensions stochastic monotonicity does not imply existence of a dual process.

(c). If X_t is both reversible and stochastically monotone, it can be shown that $d_1(t) \leq 1.3 d_2(t)$. Thus where both are applicable, the standard result (1.5) is stronger than Theorem 1.7.

(d). It would be interesting to know the weakest type of monotonicity condition on X_t which ensures that λ_2 is real.

(e). The example in Section 4 shows that stochastic monotonicity is not preserved under time-reversal of stationary processes, in general. Are there stronger monotonity properties which are preserved under time-reversal?

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