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Abstract. This paper gives a simpler proof of theorems characterizing mixtures of continuous time Markov chains, and mixtures of processes with stationary, independent increments.

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Running head. De Finetti's Theorem.

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1. Introduction. This paper gives a simpler proof of two theorems in Freedman (1963), characterizing mixtures of Markov chains in continuous time with recurrent, stable states (the stationarity condition is eliminated), as well as mixtures of processes with stationary independent increments. To state the first result, let I be a countable set--the state space. Let Ω be the set of all functions from $[0, \infty)$ to I , with the product σ -field F . Let $\{X_t\}$ be the coordinate process on Ω . The law P of an I -valued stochastic process is thus a probability on F . Fix $i_0 \in I$, the starting state.

Let Π be the set of standard stochastic semigroups P on $I_P \subset I$, such that $i_0 \in I_P$ and I_P is a single recurrent class of stable states; Π is a standard Borel space. Let P_{i_0} be the law of a Markov chain starting from i_0 and moving according to P , so P_{i_0} is a probability on F .

THEOREM. $P = \int_{\Pi} P_{i_0} \mu(dP)$ for some probability μ on Π iff:

- i) $P\{X_0 = i_0\} = 1$
- ii) $\{X_t\}$ has no fixed points of discontinuity
- iii) $P\{X_n = i_0 \text{ for infinitely many integers } n\} = 1$
- iv) For each $h > 0$, the P -law of $\{X_{nh} : n = 0, 1, 2, \dots\}$ depends only on the transition counts, in the sense of Freedman (1962) or Diaconis and Freedman (1980).

The mixing measure μ is unique.

For the second result, let I be the real line with the Borel σ -field, Ω the set of the functions from $[0, \infty)$ to I , and F the product σ -field in Ω . Again, let $\{X_t\}$ be the coordinate process on

Ω , and P a probability on F . This time, let $P \in \Pi$ be the law of a process with stationary, independent increments, starting from 0, continuous in probability. Again, Π is a standard Borel space.

THEOREM 2. $P = \int_{\Pi} P \mu(dP)$ for some probability μ on Π iff:

- i) $P\{X_0 = 0\} = 1$
- ii) $\{X_t\}$ is continuous in P -probability
- iii) for each $h > 0$, the P -law of $\{X_{nh} - X_{(n-1)h} : n = 1, 2, \dots\}$ is exchangeable.

The mixing measure μ is unique.

In both theorems, necessity is obvious, and the uniqueness of μ follows from corresponding results in discrete time. Sufficiency is proved by approximation through the binary rationals, and only h of the form $1/2^k$ are used. It is shown that conditional on a certain remote σ -field, the process is Markov (Theorem 1) or has stationary, independent increments (Theorem 2). The two proofs are very similar. That for Theorem 1 is given in section 2. The modifications for Theorem 2 are sketched in section 3, which also characterizes mixtures of Brownian motions or Poisson processes. A connection is then made with the theory of the Laplace transform, analogous to the connection between de Finetti's theorem for coin-tossing and the Hausdorff moment problem. Theorem 2 could easily be extended to processes taking values in a Euclidean space, or even a locally compact second countable abelian group, but such generalizations will not be discussed here. It is worth noting that neither theorem requires smoothness conditions on the sample paths.

2. The proof of Theorem 1. The following easy fact will be useful.

LEMMA 2.1. Let $\Sigma_0 \subset \Sigma_1 \subset \mathcal{F}$ be σ -fields in Ω . Let $Q(\omega, A)$ be an rcd given Σ_0 and given Σ_1 . If Σ is any other σ -field with $\Sigma_0 \subset \Sigma \subset \Sigma_1$, then $Q(\omega, A)$ is an rcd given Σ .

PROOF. $Q(\cdot, A)$ is Σ_0 -measurable, and therefore Σ -measurable. Likewise, for any $B \in \Sigma_1$,

$$\int_B Q(\omega, A) P(d\omega) = P(A \cap B) .$$

The display holds a fortiori for any $B \in \Sigma$. □

Turn now to the theorem. Let P satisfy the conditions. Consider $h = 1/2^k$. Let $\tau_{n,h}$ be the time of the n^{th} visit to i_0 by $\{X_{mh} : m = 0, 1, 2, \dots\}$, with $\tau_{0,h} \equiv 0$. Let $\{Y_{n,h} : n = 0, 1, \dots\}$ be the n^{th} i_0 -block in $\{X_{mh}\}$, viz., the finite string

$$\{X_{\tau_{n,h} + mh} : 0 \leq m < \tau_{(n+1),h} - \tau_{n,h}\} .$$

By convention, an i_0 -block starts at i_0 , and ends just before the next i_0 . Let F_h be the tail σ -field of the process of pairs $\{Y_{n,h}, \tau_{n,h} : n = 0, 1, 2, \dots\}$. Let $F_0 = \bigcup_h F_h$: here as elsewhere only h of the form $1/2^k$ are contemplated.

LEMMA 2.2. F_h increases as h decreases.

The easy proof is omitted.

LEMMA 2.3. Given F_h , the process $\{X_{nh}\}$ is a Markov chain with stationary transition matrix $P_{h,\omega}$ and state space $I_{h,\omega}$; here $i_0 \in I_{h,\omega} \subset I$ and $I_{h,\omega}$ is a single recurrent class. Furthermore, $P_{h,\omega}$

is F_h measurable and unique a.e.

PROOF. This follows from Diaconis and Freedman (1980), because F_h is intermediate between the tail σ -field and the exchangeable σ -field of $\{Y_{n,h}\}$: see Lemma 2.1. $\square$

LEMMA 2.4. Let $h \leq \frac{1}{2}$. Then $p_{h,\omega}^2 = p_{2h,\omega}$ a.e.

PROOF. By Lemma 2.3, given F_h , the process $\{X_{(2n)h}\} = \{X_{n(2h)}\}$ is conditionally Markov with transitions $p_{h,\omega}^2$. It remains only to show that $p_{h,\omega}^2$ is F_{2h} -measurable. But, e.g., for $i_1 \neq i_0$, $p_{h,\omega}^2(i_0, i_1)$ is the limiting relative frequency of the i_0 -blocks of $\{X_{n(2h)}\}$ which begin (i_0, i_1) , and is therefore F_{2h} -measurable. The balance of the argument is omitted. $\square$

Of course, $j \in I_{h,\omega}$ iff j is reachable from i_0 relative to $P_{h,\omega}$, i.e., $(P_{h,\omega}^n)(i_0, j) > 0$ for some n . If $j \in I_{2h,\omega}$, then $0 < P_{2h,\omega}^n(i_0, j) = P_{h,\omega}^{2n}(i_0, j)$, so $j \in I_{h,\omega}$; i.e., $I_{h,\omega} \supset I_{2h,\omega}$. In principle, $I_{h,\omega}$ could have subclasses of period 2 relative to $P_{h,\omega}$; then $I_{2h,\omega}$ would be strictly smaller; as will be seen, however, in fact this cannot happen.

LEMMA 2.5. As $h \rightarrow 0$, $P_{h,\omega}(i_0, i_0) \rightarrow 1$ in probability.

PROOF. $\int P_{h,\omega}(i_0, i_0) P(d\omega) = P\{X_h = i_0\} \rightarrow 1$ as $h \rightarrow 0$ by conditions (i-ii). $\square$

LEMMA 2.6. For P -almost all ω :

- a) $P_{h,\omega}(i_0, i_0) \rightarrow 1$ as $h \rightarrow 0$ rapidly
- b) $P_{h,\omega}(i_0, i_0) > 0$ for all h

$$c) \quad I_{h,\omega} = I_{1,\omega}$$

PROOF. Claim a) follows from Lemma 2.5; then b) is immediate, because $P_{h,\omega}(i_0, i_0) \geq P_{h',\omega}(i_0, i_0)^{h/h'}$ for $h' < h$. To prove c), let $j \in I_{h,\omega}$, so $P_{h,\omega}^n(i_0, j) > 0$ for some n . Find m such that $(n+m)h$ is an integer. Now

$$P_{1,\omega}^{(n+m)h}(i_0, j) = P_{h,\omega}^{n+m}(i_0, j) \geq P_{h,\omega}(i_0, i_0)^m P_{h,\omega}^n(i_0, j) > 0. \quad \square$$

Let $I_\omega = I_{h,\omega}$. Plainly, $\{P_{h,\omega} : h = 1/2^k \text{ and } k \text{ is a nonnegative integer}\}$ extends to a unique semigroup $\{P_{r,\omega}\}$ of stochastic matrices on I_ω , where r runs through the nonnegative binary rationals R , and $P_{0,\omega}$ is the identity matrix by convention. Recall that F_h increases to F_0 as h decreases.

LEMMA 2.7. Given F_0 , the process $\{X_r : r \in R\}$ is conditionally Markov, with stationary transitions $\{P_{r,\omega}\}$.

PROOF. This is immediate from the forward martingale convergence theorem. $\square$

The next objective is to extend $\{P_{r,\omega}\}$ to a standard stochastic semigroup on $[0, \infty)$.

LEMMA 2.8. For P -a.a. ω , for each $j \in I_\omega$, $P_{r,\omega}(j, j) \rightarrow 1$ as $r \rightarrow 0$. (The convergence need not be uniform in j .)

PROOF. Fix j and n . Given $X_n = j$, with probability 1, the process $\{X_r\}$ must stay in j on the interval $[n, n+\epsilon)$, where $\epsilon > 0$ is random. This remains true given F_0 , and the lemma follows: convergence a.e. implies convergence in probability. $\square$

LEMMA 2.9. For P -a.a. ω :

- a) $P_{\cdot, \omega}(j, k)$ is uniformly continuous for each $j \in I_\omega$, and extends to a continuous function $P_{t, \omega}(j, k)$ of nonnegative real t .
- b) $P_{t, \omega}$ is a substochastic matrix.
- c) $P_{t+s, \omega}(i, k) \geq \sum_j P_{t, \omega}(i, j) P_{s, \omega}(j, k)$.

PROOF. The argument is standard:

Claim a). From the semigroup property,

$$P_{r+s, \omega}(j, k) = \sum_i P_{s, \omega}(j, i) P_{r, \omega}(i, k)$$

So

$$P_{r+s, \omega}(j, k) - P_{r, \omega}(j, k) = [P_{s, \omega}(j, j) - 1] P_{r, \omega}(j, k) + \sum_{i \neq j} P_{s, \omega}(j, i) P_{r, \omega}(i, k)$$

Now

$$|P_{r+s, \omega}(j, k) - P_{r, \omega}(j, k)| \leq 1 - P_{s, \omega}(j, j) .$$

Claims b) and d) follow by Fatou's lemma. □

LEMMA 2.10. Fix a sequence of times $0 = t_0 < t_1 < \dots < t_n$ and states $i_0, i_1, \dots, i_n$. Let

$$A = \{X_{t_m} = i_m \text{ for } m = 0, \dots, n\}$$

Let $B \in F_0$. Then

$$(2.1) \quad P\{A \cap B\} = \int_A \prod_{m=0}^{n-1} P_{t_{m+1}-t_m, \omega}(i_m, i_{m+1}) P(d\omega)$$

PROOF. Equation (2.1) holds for binary rational t by Lemma 2.7. Now approximate real t by binary rationals. The left side of (2.1) converges by condition (ii) of the theorem; the right side, by Lemma 2.9a and dominated convergence. $\square$

We do not yet know that $\{P_{t, \omega}\}$ is a stochastic semigroup, so (2.1) does not say that $\{X_t\}$ is Markov given F_0 .

LEMMA 2.11. For each t and each $j \in I_\omega$,

$$\sum_k P_{t, \omega}(j, k) = 1 \text{ a.e. } P.$$

PROOF. Define $G_j = \{\omega: j \in I_\omega\}$ and $G_{j, n} = \{\omega: P_{1, \omega}^n(i_0, j) > 0\}$, so $G_j = \bigcup_{n=1}^{\infty} G_{j, n}$. Fix j and n . Now Lemma 2.10 shows:

$$(2.2) \quad P\{X_n = j\} = \int_{G_{j, n}} P_{n, \omega}(i_0, j) P(d\omega)$$

$$(2.3) \quad P\{X_n = j \text{ and } X_{n+t} = k\} = \int_{G_{j, n}} P_{n, \omega}(i_0, j) P_{t, \omega}(j, k) P(d\omega)$$

The sum on k of the left side of (2.3) equals the left side of (2.2); in view of Lemma 2.9b),

$$\sum_k P_{t, \omega}(j, k) = 1 \text{ a.e. on } G_{j, n}.$$

The balance of the argument is omitted. $\square$

LEMMA 2.12. For P -a.a. ω , $\{P_{t,\omega}\}$ is a standard stochastic semigroup.

PROOF. Let $H_\omega = \{t: \sum_j P_{t,\omega}(i,j) = 1 \text{ for all } i \in I_\omega\}$. By Lemma 2.11 and Fubini's theorem, for a.a. ω , the complement of H_ω is a Lebesgue-null set. On the other hand, H_ω is closed under addition, by Lemma 2.9c). Thus, $H_\omega = [0, \infty)$, i.e., $P_{t,\omega}$ is a stochastic matrix. Now sum the inequality in Lemma 2.9c) over k , to see that equality holds, i.e., $P_{t,\omega}$ is a semigroup. That $P_{t,\omega}$ is standard follows from Lemma 2.9a). $\square$

Clearly, $i_0 \in I_\omega$ and $I_\omega \subset I$ is a single recurrent class of stable states for $P_{t,\omega}$. Lemma 2.10 implies the following result, which gives the theorem.

PROPOSITION 2.1. Given F_0 , the process $\{X_t\}$ is conditionally Markov with transition $\{P_{t,\omega}\}$.

The argument really shows the existence of $E \in F_0$, such that $P(E) = 1$ for all P satisfying the conditions of the theorem, and the existence of the standard stochastic semigroup $\{P_{t,\omega}\}$ for all $\omega \in E$.

Mixtures of processes with instantaneous states can probably be characterized the same way, replacing condition (ii) by continuity in probability.

The definition of F_h may seem a bit complicated, but neither the tail σ -field nor the exchangeable σ -fields are nested. We discuss the tail σ -field in discrete time. Let $X_0, X_1, X_2, \dots$ be I -valued, starting from i_0 , with

infinitely many visits to i_0 , on even times. Let Σ_1 be the tail σ -field of the i_0 -blocks in $\{X_0, X_1, X_2, \dots\}$. Let Σ_2 be the tail σ -field of the i_0 -blocks of $\{X_0, X_2, X_4, \dots\}$. If $\Sigma_1 \supset \Sigma_2$ then atoms of Σ_1 cannot split atoms of Σ_2 . This is false by example.

Let $i_1, i_2, \dots$ be any infinite sequence in $I - \{i_0\}$. Consider the X_n -sequence

$$X_{2n} = i_0 \text{ for all } n$$

$$X_1 = i_0, \quad X_3 = i_1, \quad X_5 = i_0, \quad X_7 = i_2, \quad X_9 = i_0, \quad X_{11} = i_3, \dots$$

The i_0 -blocks of X_n are then

$$i_0, i_0, i_0 i_1, i_0, i_0, i_0 i_2, i_0, i_0, i_0 i_3, \dots$$

An atom of Σ_1 consists of all X_n -sequences whose i_0 -blocks eventually agree with this. Consider the clock time for the start of the doublets $i_0 i_j$. For an ω in our atom, this is eventually even, or eventually odd. If even, the i_0 -blocks in $\{X_{2n}\}$ end up identically $i_0 i_0 i_0, \dots$. If odd, then i_0 -blocks in $\{X_{2n}\}$ end up agreeing from some point on with $i_0 i_1, i_0 i_2, i_0 i_3, \dots$. Thus a Σ_1 -atom splits a Σ_2 -atom and $\Sigma_1 \not\supset \Sigma_2$.

3. The proof of Theorem 2. This follows the same general pattern for Theorem 1, but is much easier. For F_h , use the tail or exchangeable σ -field of $\{X_{nh} - X_{(n-1)h} : n=1,2,\dots\}$. Clearly, F_h increases as $h \downarrow 0$; call the limit F_0 . Given F_h , the differences are iid with common distribution $F_{h,\omega}$; and $F_{h,\omega} * F_{h,\omega} = F_{2h,\omega}$. Then we have a convolution semigroup $S_\omega = \{F_{r,\omega} : r \in \mathbb{R}\}$, and given F_0 , the process $\{X_r : r \in \mathbb{R}\}$ has stationary independent increments governed by S_ω . Now there is a simplification.

LEMMA 3.1. If $\{X_r : r \in \mathbb{R}\}$ has stationary, independent increments, then for each fixed real t , as $r \rightarrow t$, X_r converges a.e.

PROOF. This is well known; the restriction of the time domain to a countable set is crucial. Perhaps the simplest direct argument involves considering the martingale

$$\exp(iuX_r)E\{\exp(iuX_r)\}$$

where $\exp(x) = e^x$.

LEMMA 3.2. Let $Q(\omega, A)$ be a regular conditional P -distribution for $\{X_r : r \in \mathbb{R}\}$ given F_0 . Suppose that for each real t , as $r \rightarrow t$, $\{X_r\}$ is fundamental in $Q(\omega, \cdot)$ probability. Suppose too that $\{X_t : 0 \leq t \text{ real}\}$ is continuous in P -probability.

- a) $Q(\omega, \cdot)$ extends to F ; call the extension $\tilde{Q}(\omega, \cdot)$.
- b) $\tilde{Q}(\omega, A)$ is an rcd for $\{X_t\}$ given F_0 .

PROOF. Fix positive real times $t_1 < t_2 < \dots < t_k$ and bounded continuous functions $f_1, \dots, f_k$. Let $r_j \rightarrow t_j$ through $\mathbb{R}$. Now the $\tilde{Q}(\omega, d\omega')$ integral of $\prod_{j=1}^k f_j[X_{t_j}(\omega')]$ can be defined as

$$\lim \int \prod_{j=1}^k f_j[X_{r_j}(\omega')] Q(\omega, d\omega')$$

Integrate over $F \in F_0$ and use dominated convergence:

$$\begin{aligned}
\int_F \prod_{j=1}^k f_j[X_{t_j}(\omega')] \tilde{Q}(\omega, d\omega') &= \lim \int_F \prod_{j=1}^k f_j[X_{r_j}(\omega')] Q(\omega, d\omega') \\
&= \lim \int_F \prod_{j=1}^k f_j[X_{r_j}(\omega)] P(d\omega) \\
&= \int_F \prod_{j=1}^k f_j[X_{t_j}(\omega)] P(d\omega)
\end{aligned}$$

by the continuity assumption on P . □

Two special cases of Theorem 2 are worth considering.

1) Suppose $\{X_t\}$ has continuous sample paths. Then, Q_ω is defined in the space of sample paths on the binary rationals; it assigns measure 1 to the paths which are uniformly continuous on compacts. Thus, $\tilde{Q}_\omega$ concentrates on the continuous sample paths. And, a process with continuous paths and exchangeable increments is a scale-drift mixture of Brownian motions: indeed, a process with continuous sample paths and stationary, independent increments is a Brownian motion.

2) Suppose $\{X_t\}$ has exchangeable increments and sample paths which are counting functions. Then it is a mixture of Poisson processes.

David Aldous and Persi Diaconis remark that the last observation enables us to develop the theory of the Laplace transform, just as de Finetti's theorem for coin tossing solves the Hausdorff moment problem (Feller, 1971, p. 228). For instance, let L be defined on $[0, \infty)$. When is there a probability μ on $[0, \infty)$ such that:

$$L(t) = \int_0^\infty e^{-\lambda t} \mu(d\lambda) ?$$

Necessary conditions are that $L(0) = 1$ and L is C_∞ ; while $L' \leq 0$, $L'' \geq 0$, etc.

According to Bernstein's theorem, these conditions are also sufficient: for instance, see (Feller, 1971, p. 439). Here is a rough sketch of the idea for a probabilistic proof. Initially, we tried to construct a process

$\{X_t\}$ with exchangeable increments and counting sample functions, having $X(0) \equiv 0$ and $L(t) = \text{prob}\{X_t = 0\}$. This seemed hard to do. Instead, we made a "completely exchangeable" process of binary trees $T_0, T_1, T_2, \dots$ of random variables; the variables take two values, 0 and 1. More specifically, $T_n = \{X_{ns}\}$ where $X_{ns} = 0$ or 1 and the node ns consists of the nonnegative integer n followed by a finite string s (perhaps empty) of 0's and 1's. These T_n are required to be exchangeable. Also, each T_n splits into T_{n0} and T_{n1} , the left half and right half. These are required to be exchangeable too. And so forth. That is our definition of "completely exchangeable." Technically, T_{n0} for example is the tree $\{T_{n0s}\}$, where $0s$ denotes 0 followed by the string s .

We require that each variable be the maximum of the variables at the two successor nodes, so $X_{ns} = X_{ns0} \vee X_{ns1}$. Finally, we require

$$\text{Prob}\{1^{\text{st}} \text{ } j \text{ variables at level } k \text{ are } 0\} = L(j/2^k).$$

Here, the nodes are ordered lexicographically: the first three nodes at level 0 are 0, 1, 2; the first five nodes at level 1 are 00, 01, 10, 11, 20; and so on.

Informally, the nodes correspond to intervals, e.g., the node n corresponds to the interval $[n, n+1]$, the node n_0 to $[n, n+\frac{1}{2}]$, the node n_{01} to $[n+\frac{1}{4}, n+\frac{1}{2}]$, and so forth. The variable $X_{ns} = 0$ iff there is no dot in the corresponding interval for the desired counting process.

Formally, we can construct the tree distributions consistently down to any finite level, and then use the Kolmogorov consistency theorem to get the infinite trees. For example, for levels 0 and 1, we define $\{\xi_n: n=0,1,\dots\}$ to be exchangeable and

$$P\{\xi_0 = \dots = \xi_{N-1} = 0\} = L(N/2)$$

See (Feller, 1971, p. 228). We use the pair ξ_0, ξ_1 for x_{00}, x_{01} ; the pair ξ_2, ξ_3 for x_{10}, x_{11} . And so on. We set $x_0 = x_{00} \vee x_{01}$, $x_1 = x_{10} \vee x_{11}$. This is consistent because

$$\begin{aligned} P\{x_0 = \dots = x_{N-1} = 0\} &= P\{\xi_0 = \xi_1 = \dots = \xi_{2N-2} = \xi_{2N-1} = 0\} \\ &= L(2N \cdot \tfrac{1}{2}) = L(N) \end{aligned}$$

We now condition on the tail σ -field Σ of $\{T_n\}$. The fragments are iid trees. Clearly, T_n is a 1-1 function of (T_{n0}, T_{n1}) . So the tail σ -field of T_n equals the tail σ -field of T_{ns} , where ns is lexicographically ordered along any fixed level k , i.e., strings s of length $k=0,1,\dots$. Given Σ , we have at level k iid variables x_{ns} , which are 0 with probability $p_{k,\omega}$. Clearly, $p_{k,\omega} = p_{k+1,\omega}^2$, so $p_{k,\omega} = (p_{0,\omega})^{1/2^k}$. If $p_{0,\omega} = 0$ then $p_{k,\omega} = 0$ for all k ; then let $\lambda_\omega = \infty$, else let $p_{0,\omega} = \exp(-\lambda_\omega)$ and then $p_{k,\omega} = \exp(\lambda_\omega/2^k)$, where $0 \leq \lambda_\omega < \infty$. Now for $j \geq 1$,

$$\begin{aligned} L(j/2^k) &= P\{1^{\text{st}} \ j \text{ variables at level } k \text{ are } 0\} \\ &= \int_{\lambda < \infty} e^{-\lambda j/2^k} dP \end{aligned}$$

On $\lambda = \infty$, all variables are 1, and $L = 0$. Thus, for $t > 0$,

$$L(t) = \int_{\lambda < \infty} e^{-\lambda t} dP.$$

Let $t \rightarrow 0$, so $P(\lambda < \infty) = L(0+) = 1$. This completes the proof of the sufficiency of the condition for L to be a Laplace transform, using de Finetti's theorem for trees--but not Theorem 2. For a derivation through the Martin boundary, see Watanabe (1960).

4. Other literature. Theorem 2 goes back to Buhlmann (1960). For another exposition, see Aldous (1984, sec. 10). For a stopping-time approach to Theorems 1 and 2, see Kallenberg (1982).

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