Trend Analysis: Time Series and Point Process Problems

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ABSTRACT

The concern is with trend analysis. The data may be time series or point process. Parametric, semiparametric and nonparametric models and procedures are discussed. The problems and techniques are illustrated with examples taken from hydrology and seismology. There is review as well as some new analyses and proposals.

KEY WORDS: Biased sampling; earthquakes; floods; linear trend; non-parametric model; monotonic trend; point process; parametric model; semi-parametric model; time series; trend.

1. INTRODUCTION

The question of the presence or absence of trend in a time series or point process commonly arises in hydrological and environmental problems. Some current papers by statisticians addressing the problem include: Bloomfield (1992), Bloomfield et al. (1988), Bloomfield and Nychka (1992), Reinsel and Tiao (1987), Smith (1989). A basic question is: Just what is a trend? By way of introduction, consider Figure 1. This is a plot of the time series of water usage each month for the period 1966 to 1988 in London, Ontario, Canada. Here is a case where it would seem that few would deny that a trend is present (and also a seasonal effect). Yet setting down a precise definition is not easy. The concept of trend will be studied in this paper through examples and models. There will be parallel discussion of the time series and point process cases. There will be review of

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some existing procedures.

Point process data also arise in environmental problems. Consider Figure 2. It presents the (marked) point process of available data on large earthquakes in China from 1177 BC to 1976 AD, with the earthquake magnitudes also indicated. In this case too it would seem that a trend was present, here in the rate with which earthquakes are occurring. Again it appears difficult to set down a unique analytic definition.

A specific problem that will be addressed in the paper is whether there is a trend in the level of the Rio Negro River at Manaus, Brazil. Figure 3 provides some plots of the data. The top display graphs the time series of yearly means from 1903 through 1992. The second display correspondingly graphs the monthly means. Finally, to illustrate the general character of the daily values, the bottom display provides daily values for year 1992. The question of whether, because of deforestration, there is an an increase in flooding arises, see Sternberg (1987). An increase is claimed inevitable, but the present problem is whether or not it has yet shown itself.

Floods occur on the Rio Negro at Manaus. Their times consitute point process data. Figure 4 provides the years of floods from 1890 through 1992. The early floods of 1892, 1895, 1898 are those recorded by Le Cointe, see Sternberg (1987). For the later years a flood is defined as a river level exceeding 28.5m sometime in the year.

The layout of the paper is as follows. Section 2 comments on the general problem. Section 3 updates two time series analyses of the Rio Negro with more recent data. Section 4 studies the point process of floods. Section 5 reviews a method to handle trends arising from biased sampling. Section 6 suggests extensions. Section 7 provides some general discussion. There is an Appendix that comments on a technical detail concerning standard error estimation.

One goal of the work is to carry through the various computations employing a standard statistical package. The package employed was S, see Becker, Chambers and Wilks (1988). A second intention is to bring out the contrasting characters of parametric, semiparametric and non-parametric approaches. The distinction amongst these is: finite dimensional parameter versus finite dimensional parameter plus infinite dimensional versus only infinite dimensional. A third intention is to highlight some time series and point process similarities and differences.

There are two facets to the problem and discussion. The first is the question of whether a trend is present. The second is, assuming a trend present, how does one estimate it? These will be formalized through including a (trend) function, $S(t)$, in the models and asking: Is $S(t)$ constant? What is an estimate of $S(t)$?

2. ASPECTS OF TREND

There are various ideas associated with the notion of trend. These include: slow structural change, secular variation, drift, tendency, evolution, systematic component. Trends may further be: deterministic or stochastic, smooth or jumpy, additive or nonadditive, monotonic, modulating. A variety of approaches to trend analysis are available. These include: black box versus conceptual, descriptive versus model-based, time-side versus frequency-side, nonparametric versus semiparametric versus parametric.

Many authors have discussed the concept of trend. Perhaps the most substantial is that of Harvey (1989), Section 6.1.1. In the end even he is unable to be unequivocal and quotes Cairncross, "A trend, is a trend, is a trend ... " Recent papers considering the topic include: Cox (81), Dagum and Dagum (1988), Granger (1988), Sims (1989), El-Shaarawi and Niculescu (1992), Phillips (1991), Milbrodt (1992), Garcia-Ferrer and Del Hoya (1992). There is also a broad literature concerned with seasonal adjustment. That work typically addresses the problem of trend estimation at the same time. One reference is Kitagawa and Gersch (1984). An example of a specific technique is the procedure sabl, see Becker et al. (1988). A related problem is detecting change. Recent general references to that topic include Pettitt (1989), Lombard (1989) and Tang and McLeod (1992).

3. TIME SERIES APPROACHES

A common method of analyzing time series data is via moment parameters. In the stationary case the autocovariance function

$$c_{YY}(u) = cov\{Y(t+u), Y(t)\}$$

$u = 0, \pm 1, ,...$ is a basic parameter. It may be estimated by

$$c_{YY}^{T}(u) = \frac{1}{T} \sum_{t=0}^{T-|u|} [Y(t+u)-\bar{Y}][Y(t)-\bar{Y}] \tag{1}$$

with $\bar{Y}$ the mean of the data vaules $Y(t)$, $t = 0, ..., T-1$. Trend can show itself in the values of the sample autocorrelation function $c_{YY}^{T}(u)/c_{YY}^{T}(0)$ being near 1 for small u. In this connection see Figures 5 and 6. These are based on the Rio Negro data. Figure 5 graphs the annual maximum, mean and minimum temperatures. The horizontal dashed lines give the respective overall averages. Figure 6 provides the estimated autocovariance curves defined by (1) and approximate ± 2 standard error limits (assuming white noise). Examination of the values at small lags, u, does not suggest the presence of a trend. The graph with the greatest suggestion of an effect is the minimum. This may be associated with the higher than average values of that series from 1970 on.

Generally, likelihood based approaches may be anticipated to be more efficient than moment based in analyzing time series data. The likelihood may be set up through a family of conditional distributions, eg. via expressions for the

$$Prob\,\{y < Y(t+1) < y+dy \mid H_t\}$$

$H_t = \{Y(s),\, s \le t\}$ being the history of the process. This is particularly simple in the case that the process $Y(t)$ is Gaussian.

Likelihood-type analyses may also be set up via Fourier inference. For example, consider the model

$$Y(t) = \alpha + \beta t + \mathrm{E}(t) \tag{2}$$

$t = 0, \pm 1, \pm 2, \cdots$ with $\mathrm{E}(t)$ a 0-mean, stationary, mixing time series. The hypothesis of no trend is now $\beta = 0$. To study this hypothesis first denote the power spectrum of $E(t)$ by

$$f_{\mathrm{EE}}(\lambda) = \frac{1}{2\pi}\sum_u c_{\mathrm{EE}}(u)\, e^{-i\lambda u}$$

$-\infty < \lambda < \infty$. Consider then the Fourier transform values

$$\varepsilon_j = \sum_{t=0}^{T-1} \mathrm{E}(t) \exp\{-2\pi i j t / T\}$$

for $j = 1, \ldots, J$. Under broad conditions, see Good (1963), Akaike (1964), Duncan and Jones (1966), Hannan (1967), Brillinger (1973), these are approximately independent and satisfy a central limit theorem. In particular the variate ε_j is asymptotically normal with mean 0 and variance $2\pi T\, f_{\mathrm{EE}}(2\pi j/T)$. Further for J finite, the values $\varepsilon_1, \cdots, \varepsilon_J$ are asymptotically independent. Taking the Fourier transform of each side of the model (2) gives

$$y_j = \beta\gamma_j + \varepsilon_j \tag{3}$$

for $j = 1, \cdots, J$ where

$$\gamma_j = \sum_{t=0}^{T-1} t \exp\{-2\pi i j t / T\}$$

is the Fourier transform of the trend. The step forward here is that the model (3) is basically a simple linear of regression with independent normal errors. Assuming $f_{EE}(2\pi j/T) \approx f_{EE}(0)$, classical regression results are available. For example, the estimates

$$\hat{\beta} = \mathrm{Re}\{\sum y_j \bar{\gamma}_j \,/\, \sum |\gamma_j|^2\}$$

and

$$\hat{\sigma}^2 = 2\pi T \hat{f}_{EE}(0) = \frac{2}{2J-1} \sum |y_j - \hat{\beta}\gamma_j|^2$$

are approximately independent normal and chi-squared respectively. For the hypothesis $\beta = 0$, of no trend, a test statistic is

$$\hat{\beta}\sqrt{2\sum|\gamma_j|^2}/\hat{\sigma}$$

It is distributed approximately as a Student's t with $2J-1$ degrees of freedom under the null hypothesis. (The approach is essentially equivalent to the Whittle (1954) gaussian estimation procedure taking the $|\varepsilon_j|^2$ to be independent exponentials.)

The two-sided prob-values obtained for the three series of Figure 5, with $J = 9$ and $T = 90$ here are: for the mean 8.36%, for the maximum 23.14% and for the minimum 3.01%. For the minimum, the value is noteable. Examination of the time series plot of the minimum in Figure 7, as indicated earlier, does suggest a rise during the later years. The results presented here update those of Brillinger (1988). This approach extends directly to a trend $S(t\,|\,\theta)$ including a finite dimensional parameter θ.

The analysis provided here developed directly from central limit theorem results. These are standard for mixing (or short memory) processes possessing moments. There has been recent consideration of the long-memory case, see Yajima (1989). The distribution of the regression estimates is altered, Yajima (1991).

The model just considered was semiparametric, involving a parametric trend, $\alpha + \beta t$ and a smooth error spectrum $f_{EE}(\lambda)$. Consideration now turns to some fully nonparametric models. Consider the model

$$Y(t) = S(t) + E(t) \tag{4}$$

assuming $S(t)$ to be smooth. In this case $S(t)$ may be estimated by simply smoothing the data $Y(t)$, $t=0,...,T-1$. The results of employing a running mean of 15 are provided in Figure 7. The issue of whether a trend is present may be addressed by setting a confidence band about the overall mean level of the series. The figure shows ± 2 standard error limits. These are produced as follows: Suppose a running mean

$$\bar{Y}(t) = \frac{1}{2V+1} \sum_{v=-V}^{V} Y(t+v)$$

is computed. Then, with $A(\lambda) = \sin((2V+1)\lambda/2) \,/\, (2V+1)\sin(\lambda/2)$

$$\begin{aligned} var\ \bar{Y}(t) &= \int |A(\lambda)|^2 f_{EE}(\lambda) d\lambda \\ &\approx 2\pi f_{EE}(0) \,/\, (2V+1) \end{aligned}$$

so $f_{EE}(0)$ needs to be estimated. This may be done as in Brillinger (1989). First the residual series, $\hat{E}(t) = Y(t) - \bar{Y}(t)$, is computed. Then

$$\hat{f}_{EE}(0) = \sum_{j=1}^{J} I^T_{\hat{E}\hat{E}}\left(\frac{2\pi j}{T}\right) \Big/ \sum_{j=1}^{J} \left|1 - A\left(\frac{2\pi j}{T}\right)\right|^2$$

for moderate J where I^T denotes the periodogram. Again the noteworthy case is that of the series of minimum values. A simultaneous confidence band might also be presented. In the case of $E(t)$ white noise such a band is developed in Bjerve et al. (1985). In the stationary case, results of Leadbetter et al. (1983) may be employed. Such a band is not presented here because of concern with the accuracy of the asymptotic results in the finite case.

Another type of nonparametric analysis of the Rio Negro data may be developed as follows. Let $S(t) = E\{Y(t)\}$ denote the mean level of the series $Y(t)$ at time t. There are circumstances in which one views that a trend, here $S(t)$, is necessarily monotonic, see Granger (1988), Brillinger (1989). One can seek a test statistic that is sensitive to departure from constant to monotonic mean level. Abelson and Tukey (1963) considered this problem in the case that the observations were independent. They sought linear combinations

$$\sum_{t=0}^{T-1} c(t)Y(t)$$

that minimax a correlation coefficient. The values found were

$$c(t) = \sqrt{t(1-\frac{t}{T})} - \sqrt{(t+1)(1-\frac{t+1}{T})} \tag{5}$$

for large T. This function is graphed of Figure 8. It is seen to strongly contrast the early and late values. The extreme correlation is provided by a step function with one jump. A standardized test statistic, for stationary E(t), is provided by

$$\sum c(t)Y(t) \;/\; \sqrt{2\pi\hat{f}_{EE}(0) \sum c(t)^2} \tag{6}$$

see Brillinger (1989). Expression (6) involves an estimate of the power spectrum at frequency 0. The estimate $\hat{f}_{EE}(0)$ is determined as above. The (z-)values of the statistic (6) are -.65, 2.10 and 5.11 respectively for the maximum, mean, minimum. The two-sided prob-values are 51.3%, 3.6% and 0.0% respectively. There is very strong indication of a change in level of the minimum.

There are some general approaches to estimating "smooth" functions. One is smoothness priors / penalized log likelihood. For example, one seeks $S(t)$ to minimize

$$\sum_t [Y(t)-S(t)]^2 + \Lambda \sum_t [S(t)-2S(t-1)+S(t-2)]^2$$

The second term here enforces smoothness on $S(t)$. The parameter Λ may be estimated by cross-validation, see Gersch (1992) or by Bayesian arguments, see Akaike (1980).

A state space approach is an alternate way to proceed, see Harvey (1989), Section 2.3.2 . Here, for example one assumes a random "slope", $S_1(t)$, evolving in accord with

$$S_1(t) = S_1(t-1) + \varepsilon_1(t)$$

with $S(t)$ of (4) then given by

$$S(t) = S(t-1) + S_1(t-1) + \varepsilon(t)$$

If ε and ε_1 are identically 0 then $S(t) = \alpha+\beta t$ as already discussed. Alternatively, the hypothesis of no trend corresponds to $S_1(t)$ identically 0.

Fully parametric models that have been used in trend analysis include: ARIMA, state space, and polynomial in t plus ARMA, see for example, Harvey (1989). There is a frequency-side variant of the moving window technique, see Brillinger and Hatanaka (1969). A Fourier approach is employed by Kunsch (1986) to distinguish monotonic trend from long-range dependence.

4. POINT PROCESS APPROACHES

There is a moment based approach to the analysis of point process data. An important moment parameter is the autointensity function. Suppose that $N(t)$ counts the number of points in the interval $[0,t)$ of the process and let $dN(t)$ be one if there is a point in the small interval $[t,t+dt)$ and zero otherwise. In the stationary case one defines the autointensity, $h_{NN}(u)$, at lag u via

$$h_{NN}(u)\,du = Prob\,\{dN(t+u) = 1 \mid event \; at \; t\}$$

This parameter is analagous to the autocovariance function, but it has a much more direct interpretation. If the points $0 \le \tau_1 < \tau_2 < \cdots < \tau_{N(T)} < T$ of the process N are available, then $h_{NN}(u)$ may be estimated by

$$h_{NN}^T(u) = \#\{\,|\tau_k - \tau_j - u| < b\,\} \,/\, 2bN(T)$$

for small binwidth b. This estimate is essentially the histogram of the available $\tau_k - \tau_j$.

Figure 4 provides the point process of floods for the Rio Negro for the period 1890 to 1992. Figure 9 provides the corresponding estimated autointensity function. Also included are approximate ± 2 standard error limits. Values at lag u near 0 provide little evidence for trend.

As is the case for time series, likelihood approaches may be expected to be more efficient if the assumptions are satisfied. In the point process case the likelihood is developed from the conditional intensity function

$$Prob\,\{dN(t) = 1 \mid H_t\}$$

where H_t gives the history of the process, $\{N(s),\, s < t\}$ Snyder (1975) and Ogata and Katsure (1986) are pertinent references.

Sometimes time series procedures may be employed to analyse point process data. Specifically a time series may be set up corresponding to the point process. Let b denote a small interval size. Define the 0–1 time series N_t to be 1 if there is a point in the interval $[t, t+b)$ and to be 0 otherwise for $t = 0, \pm b, \pm 2b, \cdots$. Now one can develop a probit type analysis, for example by setting

$$\pi_t = Prob\,\{N_t = 1 \mid H_{t-1}\}$$

with $H_t = \{N_s,\, s \le t\}$ and then assuming for example

$$\pi_t = \Phi\left[S(t) + \sum_{u=1}^{U} a(u) N_{t-u} \right] \quad (7)$$

where Φ is the normal cumulative. The likelihood is

$$\sum [N_t \log \pi_t + (1-N_t) \log(1-\pi_t)]$$

The model is of autoregressive-type. In the case that $S(t)$ is parametric, eg. $S(t) = \alpha+\beta t$, references to such models include: Cox (1970), Brillinger and Segundo (1979), Kaufmann (1987), Zeger and Qaqish (1988). The Appendix contains an indication of the theory involved. A state space form could also be developed.

The table below presents the results of fitting the model (7) with $S(t) = \alpha+\beta t$ by the procedure glm of Chambers and Hastie (1992).

	estimate	s.e.
$\hat{\alpha}$	-.923	.340
$\hat{\beta}$	-.00187	.00533
$\hat{a}(1)$	.469	.350
$\hat{a}(2)$	.153	.358
$\hat{a}(3)$	-.090	.370
$\hat{a}(4)$	.273	.362
$\hat{a}(5)$	-.090	.371

There is no real indication of trend, ($\beta \neq 0$), or of time lags being necessary, provided by this analysis.

The "trend" function $S(t)$ of (7) could also be nonparametric, eg. simply assumed smooth. Estimation can then be via a running window technique. Figure 10 presents the result of such an analysis. The top two displays are the results of fitting the model (7) with no lagged values of N_t. The bottom two displays include $U = 5$ lagged values. The estimates of the $a(u)$ were insubstantial in this last case. These computations were carried out via the procedure gam of Chambers and Hastie (1992).

In the smooth case one might alternatively estimate $S(t)$ via a penalized log likelihood such as

$$\sum_t [N_t \log \pi_t + (1-N_t) \log (1-\pi_t)] + \Lambda \sum_t [S(t)-2S(t-1)+S(t-2)]^2$$

The second term imparts the smoothness to $S(t)$.

In summary, in a search for trends parametric, nonparametric and semiparametric analyses are available for the analysis of point process data as was also the case for time series. Further parametric analyses to consider include: renewal, autoregressive-like and state space. The nonparametric include: penalized likelihood and locally weighted parametric. The semiparametric include: thinned (see next section), modulated and those with systematic component. Lewis and Robinson (1973), Pruscha (1988), Gamerman (1992), Ogata and Katsura (1986) are pertinent references to other work.

5. BIASED SAMPLING

Figure 2 provides the historical record of large earthquakes in China from 1177 BC to 1976 AD. The top left panel of Figure 11 gives the running rate of events based on the available data, for the years 1000 AD to 1976 AD employing a window of width 50 years. The trend of Figure 2 remains apparent. It appears that the rate of events has been increasing fairly steadily, i.e. that a trend is present. It is difficult to think that such a substantial change is actually real. Other explanations need to be sought. On reflection, the data are the events that have been *recorded*. In the early years the information would have been passed on in irregular manners, particularly until printing was commonly available. In Lee and Brillinger (1979) a conceptual model was built to handle this circumstance.

It would seem that the chance of an earthquake making its way into the record would depend on the population (able to take note of it) and the stage of development of society. Ibid these two are handled by defining probability functions. Consideration is restricted to the data from 1000 AD on. The advent of printing occurred in the 10th century and became widespread in the 15th, hence one component of the probability function is taken to be

$$\pi_1(t) = 0.1 \; at \; t = 1000$$
$$= 1.0 \; at \; t = 1500$$

The second component is a function of population taken to be

$$\pi_2(t) = \min\{1 - [S(t) - P(t))/S]/3, 1\}$$

with $P(t)$ the taxation census in year t and $S = 50 \; million$. The overall probability of an earthquake making its way into the record is then taken as

$$\pi(t) = \pi_1(t)\pi_2(t)$$

This function may be employed to correct for the biased sampling that has taken place. The graph on the top right of Figure 11 provides the $\pi(t)$ employed. (The dip after 1600 relates to wars and dynasty change.)

Suppose the actual point process of earthquake occurences is $M(t)$. Then the observed point process $N(t)$ can be viewed as the result of thinning $M(t)$ with $\pi(t)$ giving the probability that an event that occurred on date t is actually included in the data set. The rate of the process $N(t)$ at time t is then $E\{dN(t)\}/dt = \pi(t)p_M$ where p_M is the rate of the background stationary process $M(t)$. The bottom left figure is the "corrected" form of the top left figure, obtained by weighting inversely to $\pi(t)$. The curve now has a more stationary appearance. Finally a "corrected" autointensity is given in Figure 11 bottom right, computed as in ibid. . There is a suggestion of clustering of events.

The above technique is furthed developed and extended in Guttorp and Thompson (1991). Alternately assuming the basic process is homogeneous Poisson, for example, a likelihood analysis may be developed, see Veneziano and Van Dyck (1987).

6. EXTENSIONS

Extensions are available to various situations and sometimes something new appears. In particular, in the case of vector-valued series, random effect models become appropriate (see Shumway (1971), Brillinger (73), Bloomfield et al. (83)), with the trend able to be viewed as a common component. Point processes and time series are particular cases of processes with stationary increments, so too are marked point processes. So the theory of processes with stationary increments can suggest appropriate techniques for either time series or point processes. One can consider extensions to other data types, such as marked point processes with ordinal-valued marks. One needs models to include covariates. (The variate t is a covariate in the discussion of trend in the nonstationary case.) One can consider spatial (eg. Cressie (1986)) or spatial-temporal cases. One can consider nonlinear systems.

7. DISCUSSION

The paper has presented some corresponding techniques for trend analysis in the time series and point process cases. The techniques may be classified as: parametric, semiparametric and nonparametric. Concepetual models are important. Some form of constancy seems necessary. The estimation of uncertainty is often critical, eg. to address the issue of whether a trend or change is present. When a trend is "found" there remains a need for a scientific explanation.

In the end the definition of trend appears to depend on the circumstance. Trend seems to be an example of what Tukey calls a vague concept. To quote from Mosteller and Tukey (1977), "Effective data analysis requires us to understand vague concepts, concepts that may be made definite in many ways." This paper has illustrated several ways to make the concept specific in particular cases, but it is clear that many cases remain.

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APPENDIX

The standard errors listed in the Table of Section 4, and those later used to rule out lags when the nonparametric estimate of $S(t)$ is obtained, are those produced by the programs glm and gam. Justification is needed for their use in this time series case.

Kaufmann (1987) develops asymptotic properties of the maximum likelihood estimates of a model (3.2), of which the model (7) with

$S(t) = \alpha+\beta t$ is a particular case. Under regularity conditions Kaufmann finds the estimates to be asymptotically normal with a particular covariance matrix. Let π_t' denote the derivative of π_t and set $\mathbf{Z}_t^\tau = (1, N_{t-1}, \cdots, N_{t-U}, t)$, then that covariance matrix is the inverse of

$$\sum_{U+1}^{T} \mathbf{Z}_t \frac{(\pi_t')^2}{\pi_t(1-\pi_t)} \mathbf{Z}_t^\tau$$

Kaufmann remarks that this last becomes a consistent estimate when the parameters are replaced by their maximum likelihood estimates.

In McCullagh and Nelder (1989) the expression given for the covariance matrix is the inverse of

$$\mathbf{X}^\tau \mathbf{W} \mathbf{X}$$

where $\mathbf{X}$ is the matrix of exogenous variables and

$$\mathbf{W} = diag\,\{(\frac{d\pi_t}{d\eta_t})^2 \,/\, \pi_t(1-\pi_t)\}$$

One quickly checks that the two results are the same with the choice $\mathbf{X}_t = \mathbf{Z}_t$.

One technical difficulty is that not all of Kaufmann's assumptions are satisfied in the present case. The conditions of his Corollary 4 do hold except for the requirement that the exogenous variables, here 1 and t, be bounded. The corresponding time series results, see Hannan (1979), give hope that this requirement may be replaced by the sort of assumption Hannan makes.

Figure Legends

Figure 1. Monthly water use, expressed as a daily rate, for London, Ontario, Canada.

Figure 2. Historical earthquakes in China, by date and magnitude.

Figure 3. The level of the Rio Negro as recorded at Manaus, Brazil daily since 1903.

Figure 4. Flood years for the Rio Negro at Manaus. The lower graph provides the cumulative count since 1890.

Figure 5. Plots of the annual maximum, mean, minimum stage as computed from daily values.

Figure 6. Estimated autocovariances of the series of Figure 5. Approximate ± 2 white noise standard error limits have been added.

Figure 7. Running means of length 15 years of the three series of Figure 5. The dashed lines give ± 2 standard errors.

Figure 8. The Abelson-Tukey coefficients of (5).

Figure 9. The estimated autointensity of the Rio Negro flood point process of Figure 4.

Figure 10. The top two displays are $\hat{S}(t)$ and $\hat{\pi}_t$ of model (7) with no lagged N values. The bottom two are the corresponding quantities when $U = 5$.

Figure 11. The upper left display is the running rate of recorded events with a window of 50 years. The upper right is the estimated probability function (7). The lower left is the corrected series. The lower right is the estimated autointensity.

Figure 1: Monthly water use London, Ontario

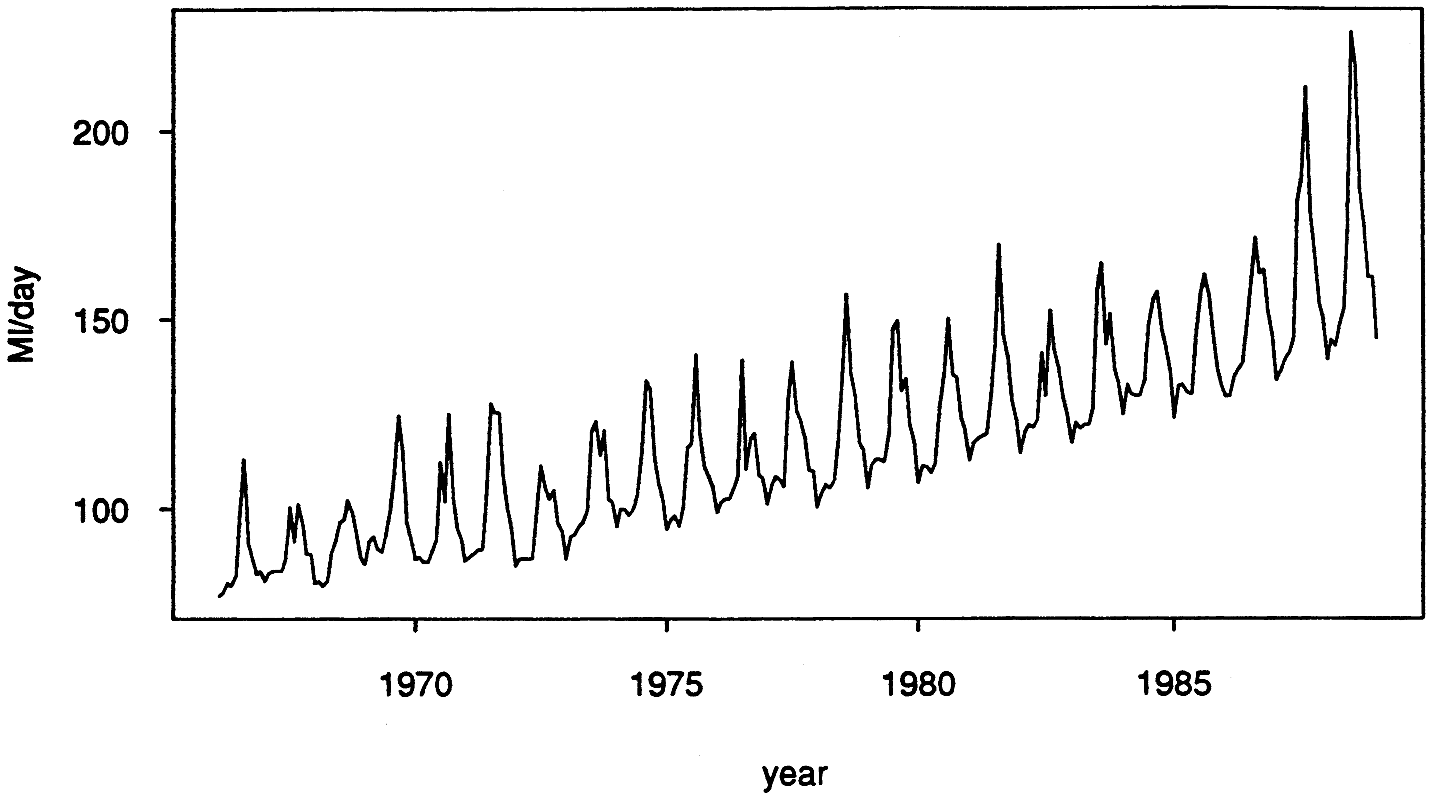
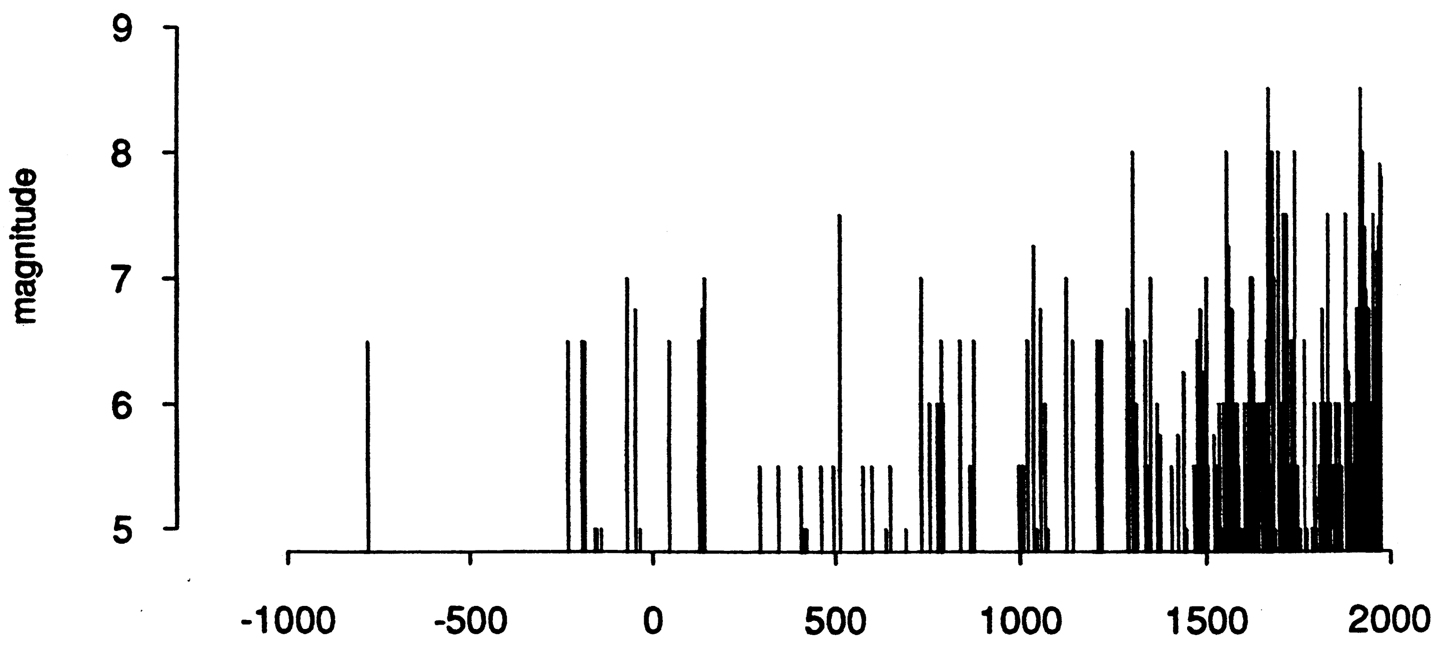


Figure 2: Chinese earthquakes 1177 BC - 1976 AD



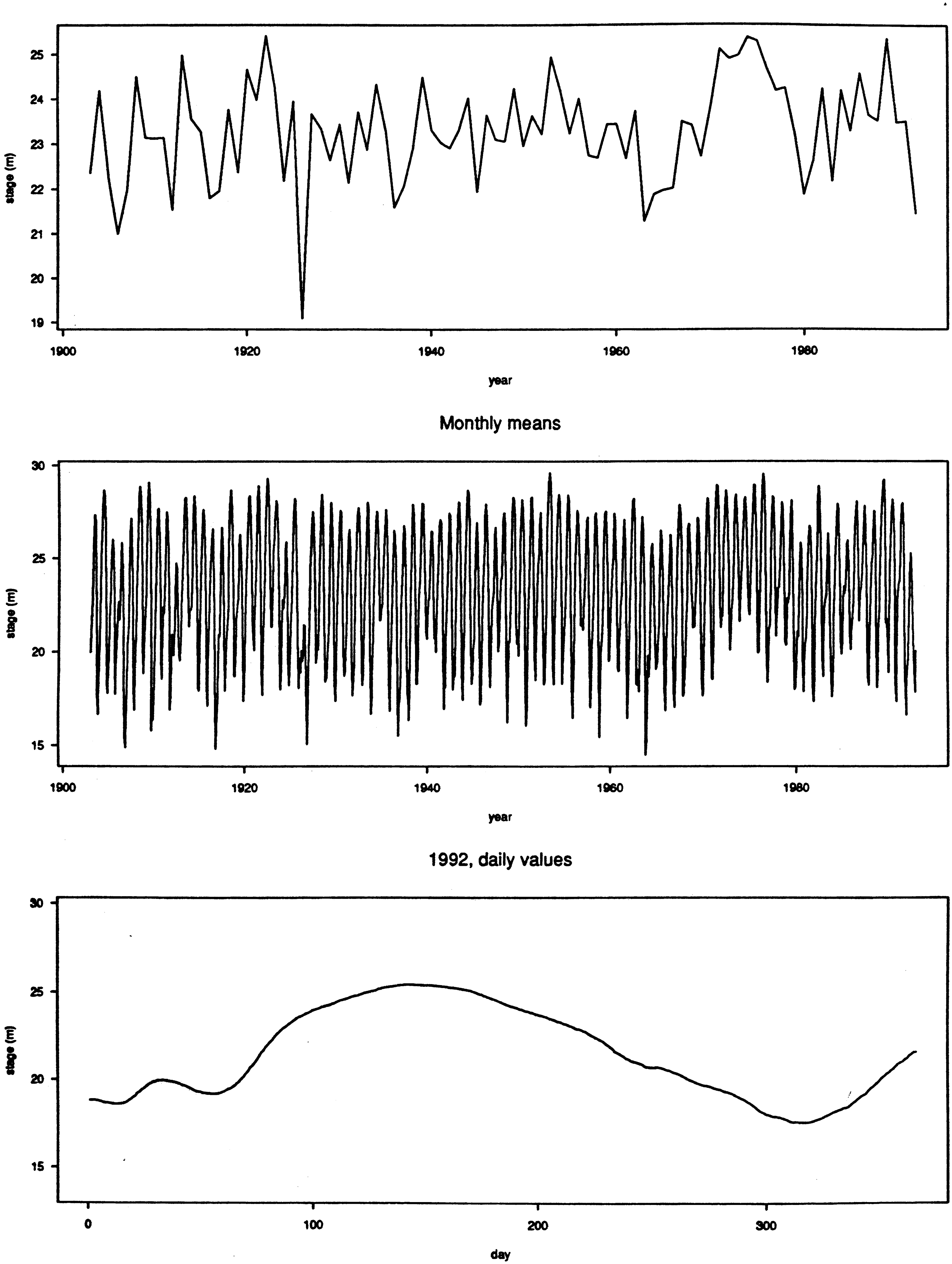
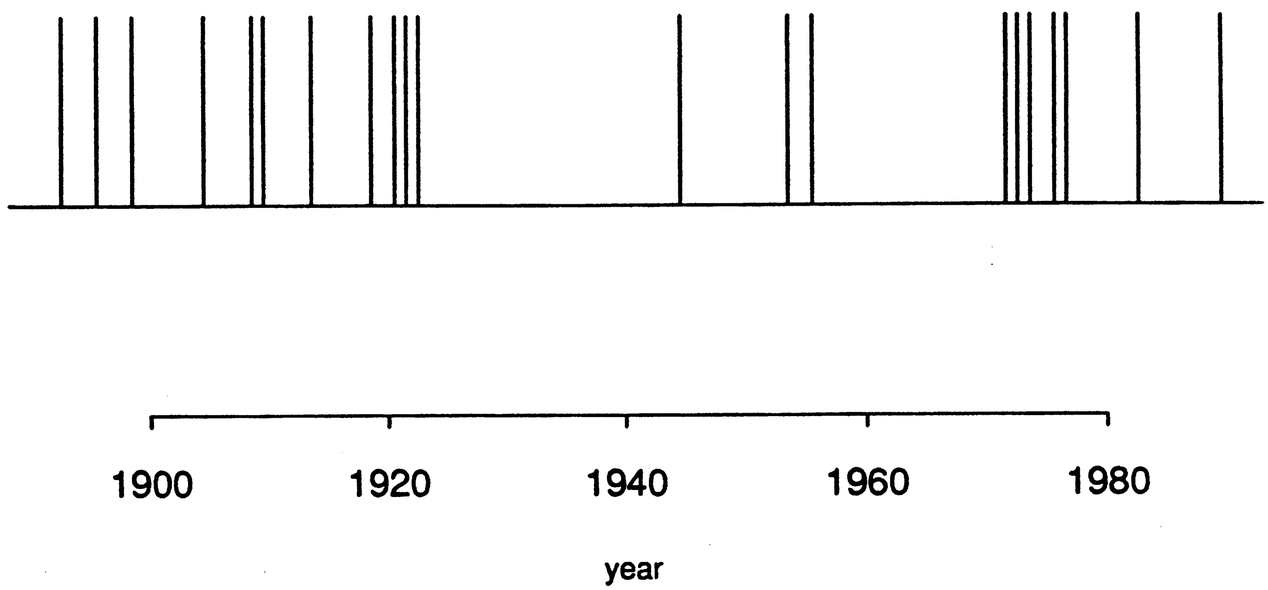

stage (m)
year
Monthly means
stage (m)
year
1992, daily values
stage (m)
day

Figure 4: Rio Negro floods 1892-1992



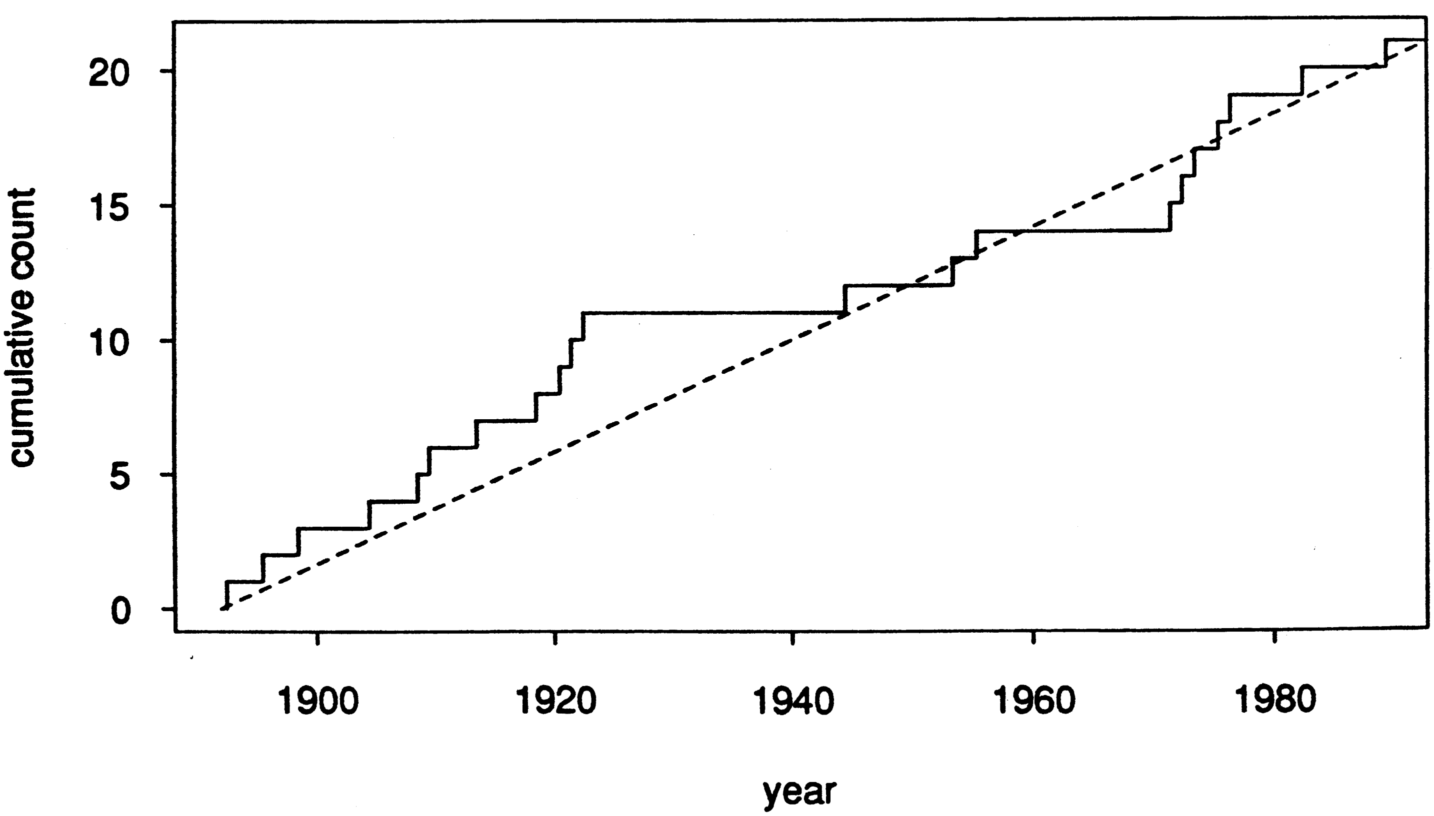


Figure 5: Annual Maximum, Mean, Minimum Stages at Manaus

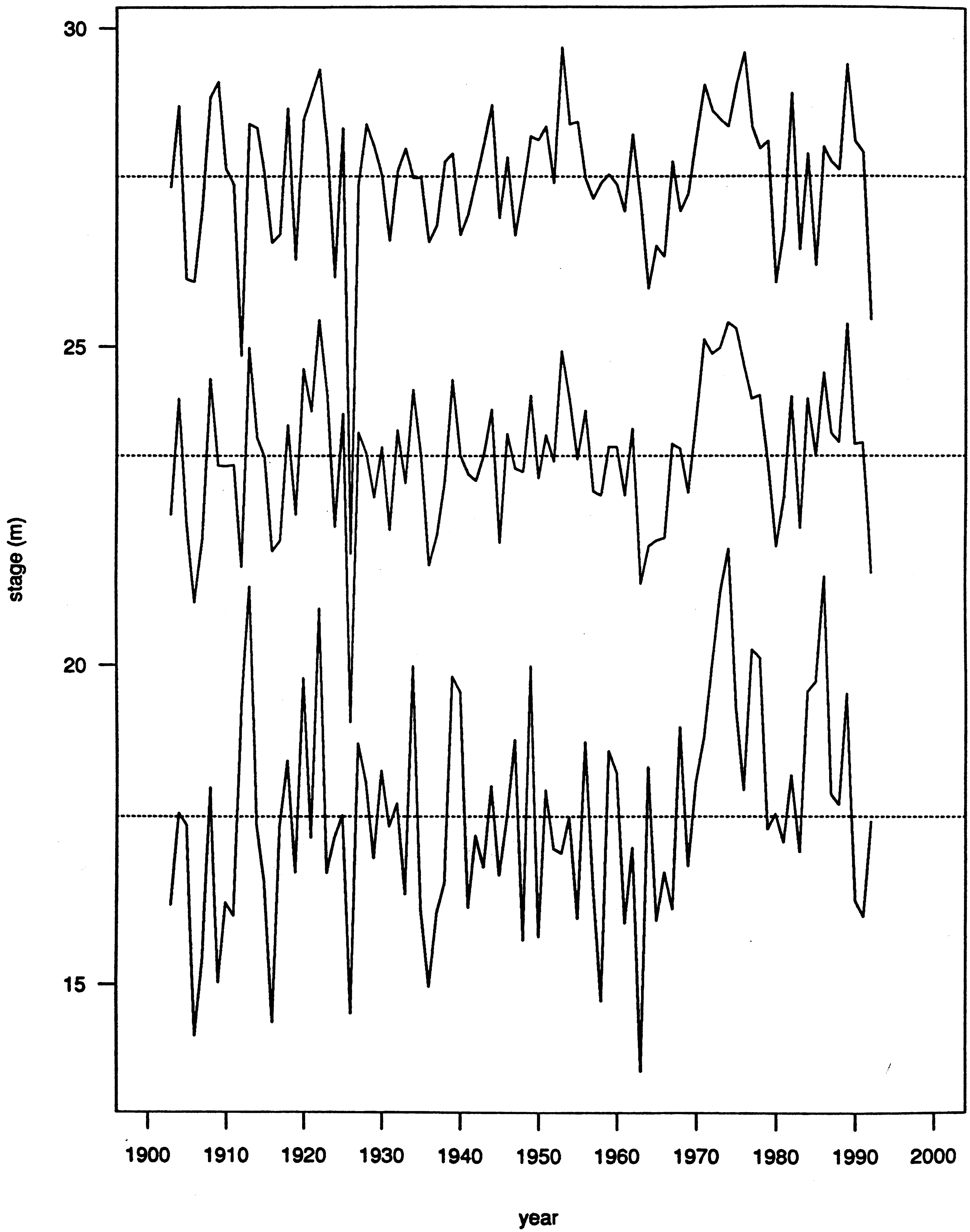
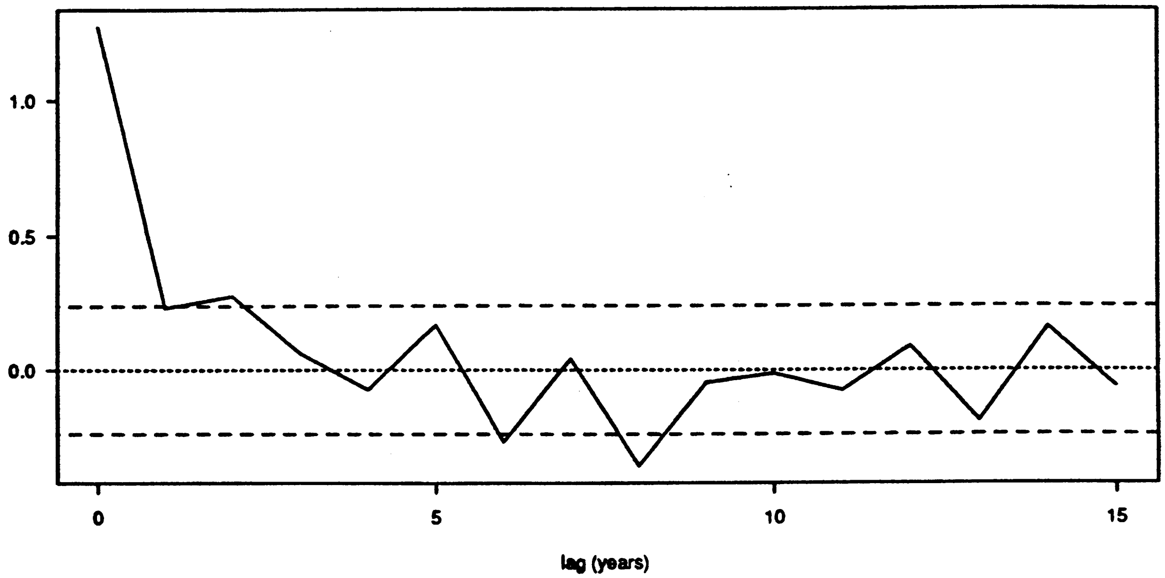
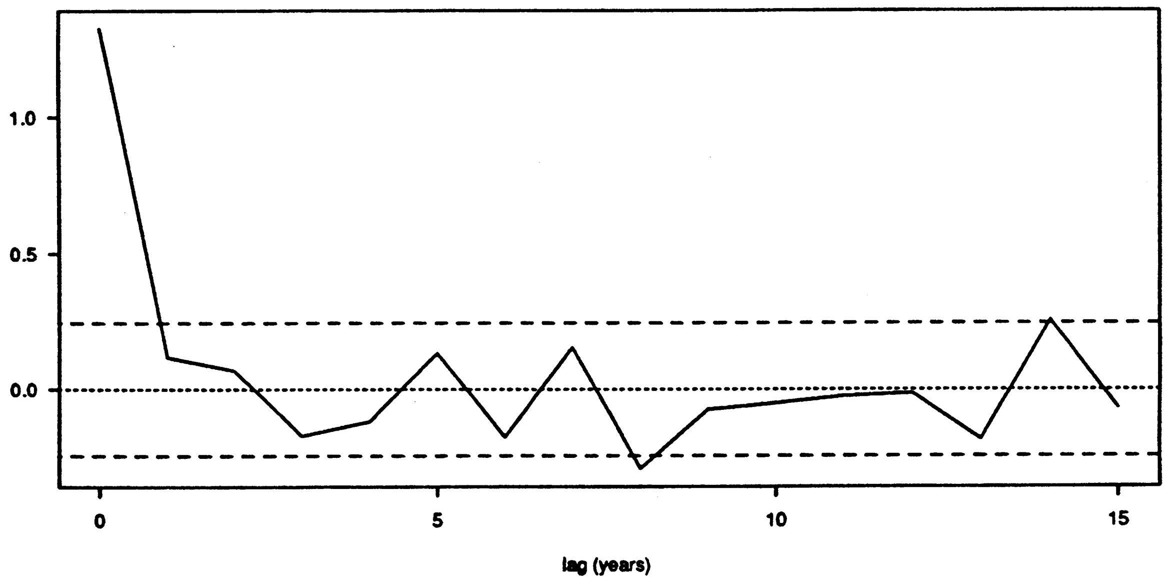


Figure 6: Autocovariance yearly means



Autocovariance yearly maxima



Autocovariance yearly minima

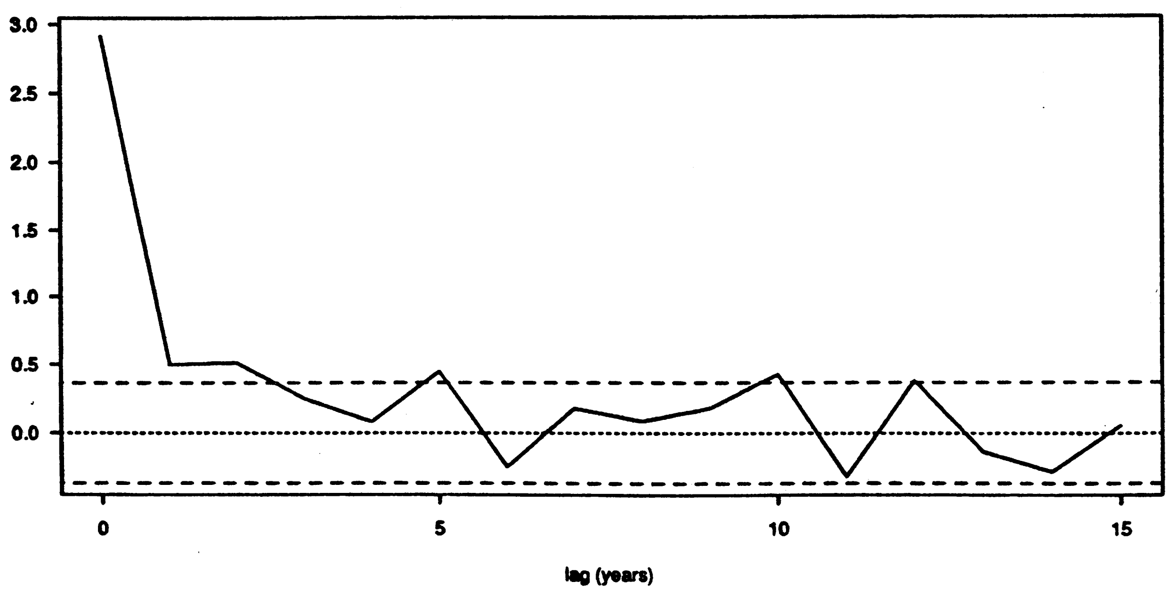


Figure 7: Smooth 'trends'

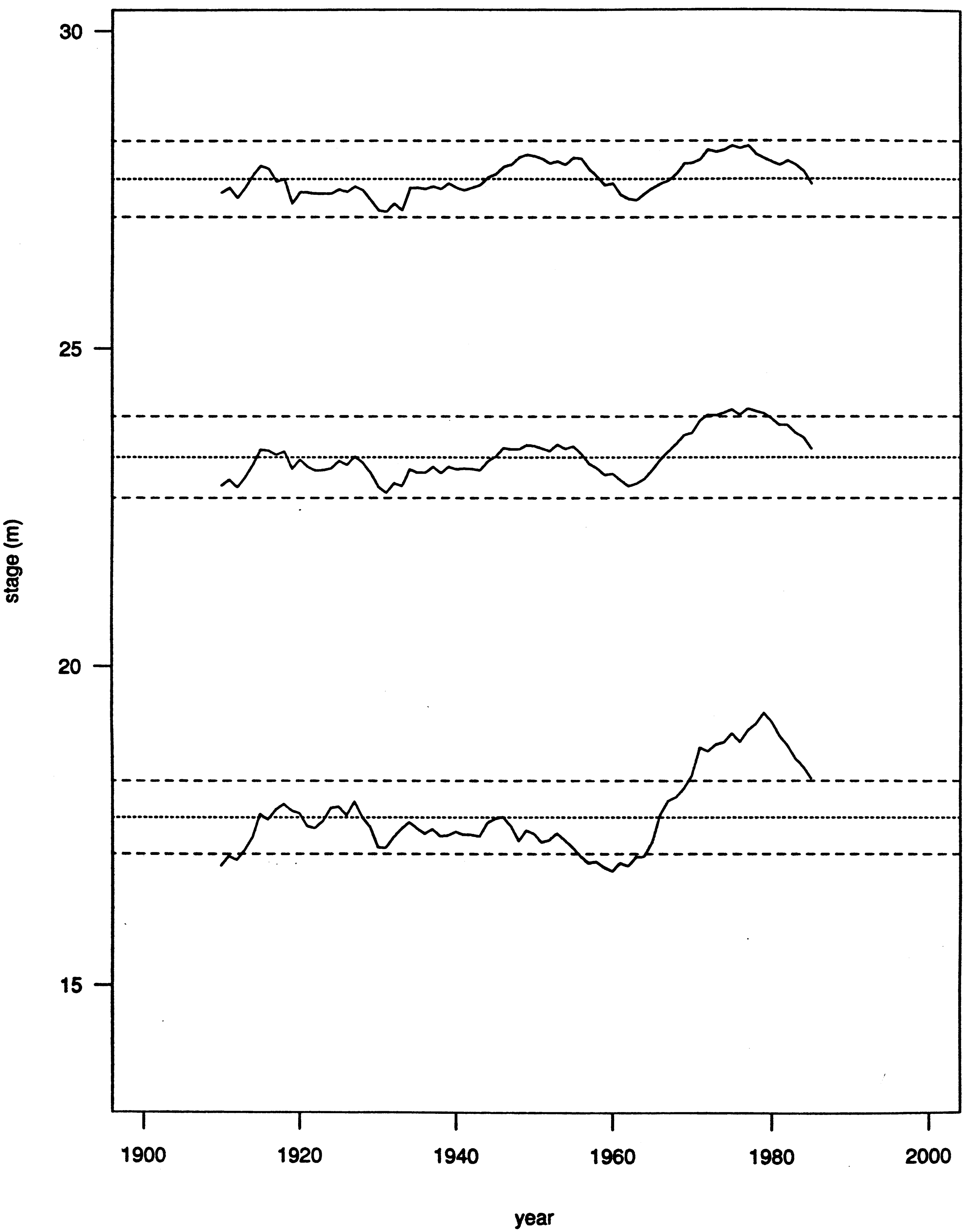


Figure 8: Abelson-Tukey coefficients

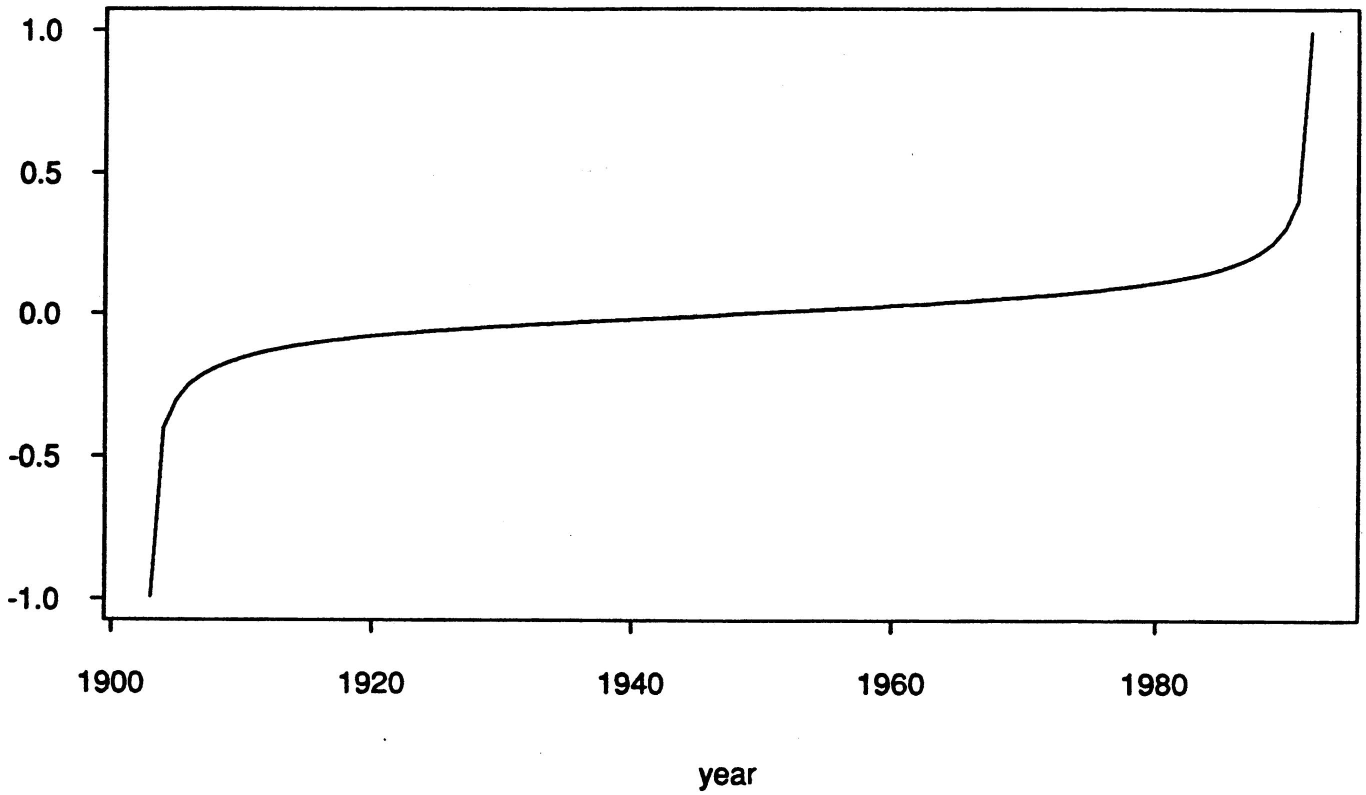


Figure 9: Autointensity Rio Negro floods

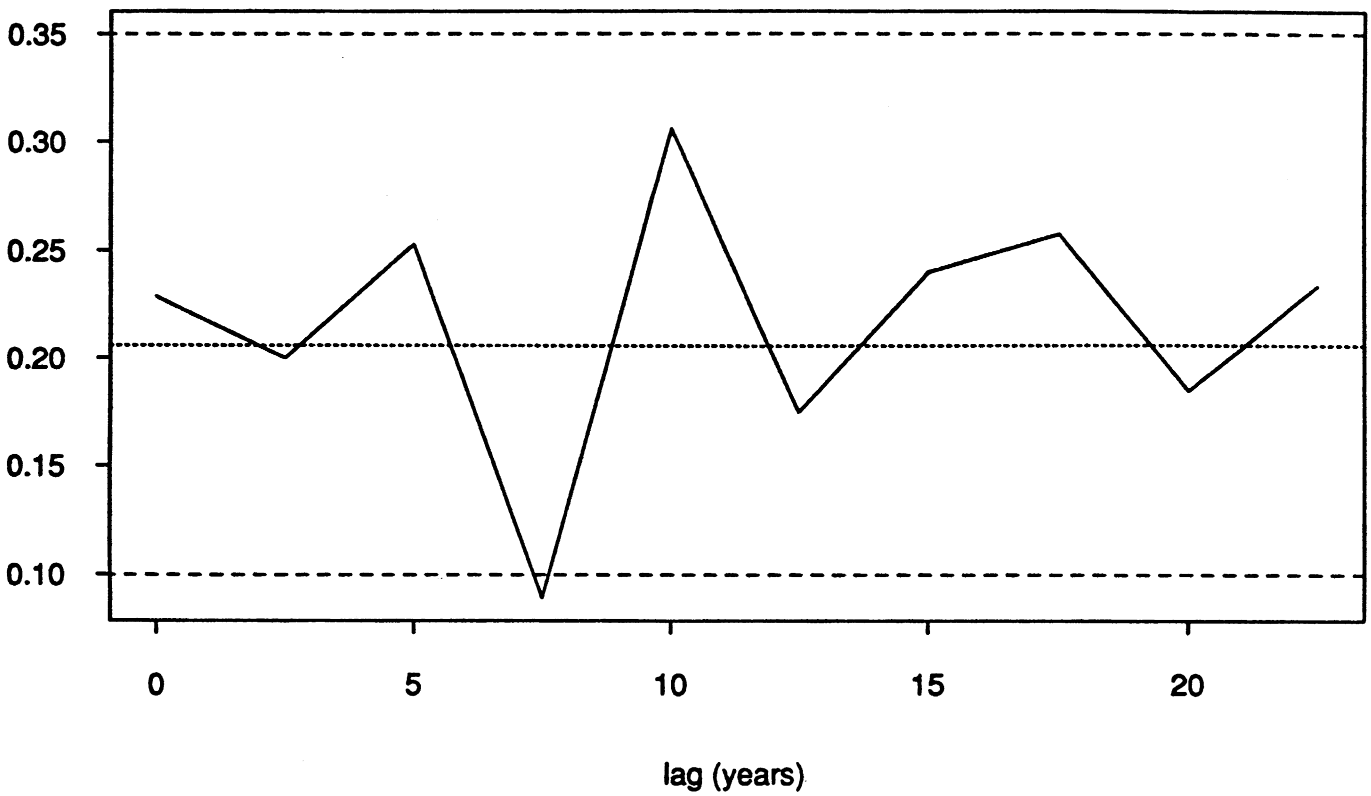
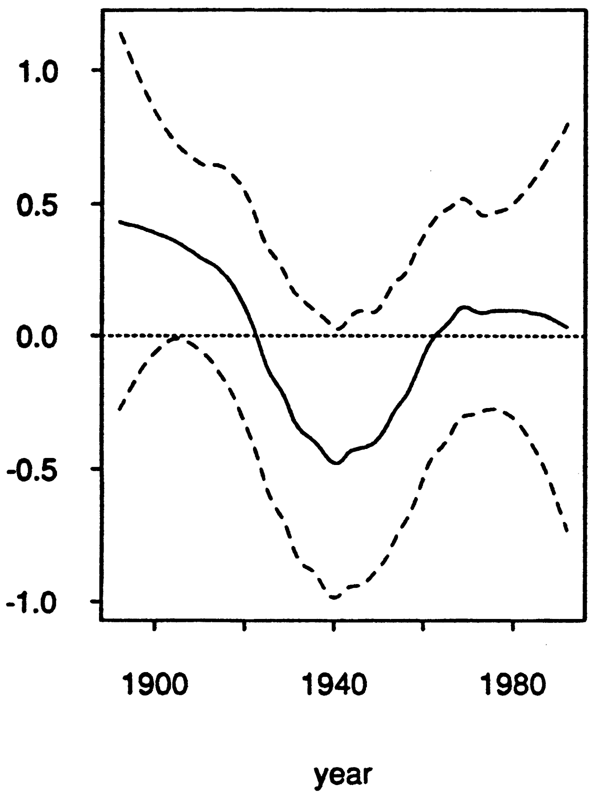
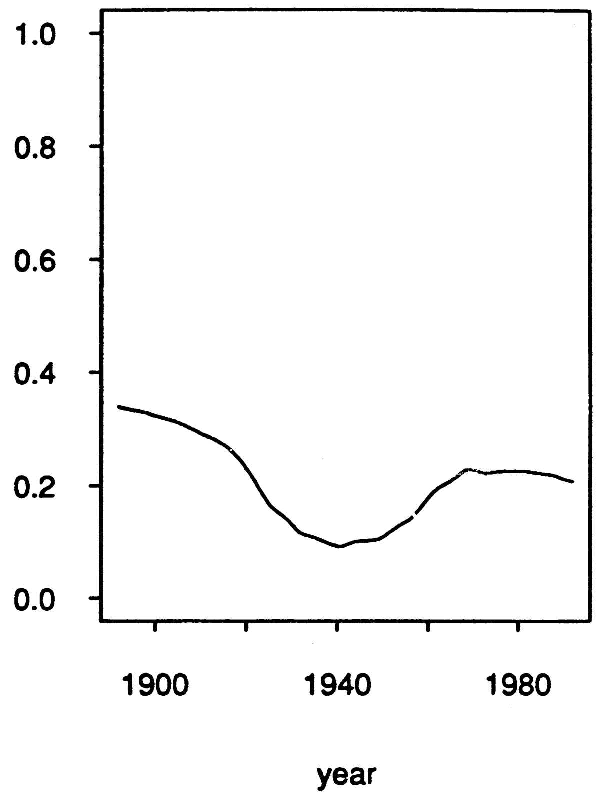


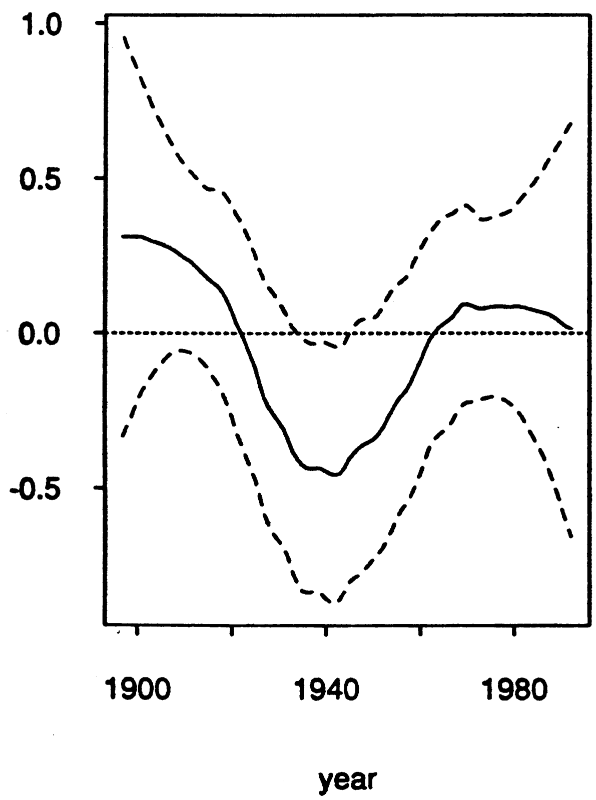
Figure 10: Smooth 'trend'



Fitted probability



Smooth 'trend'



Fitted probability

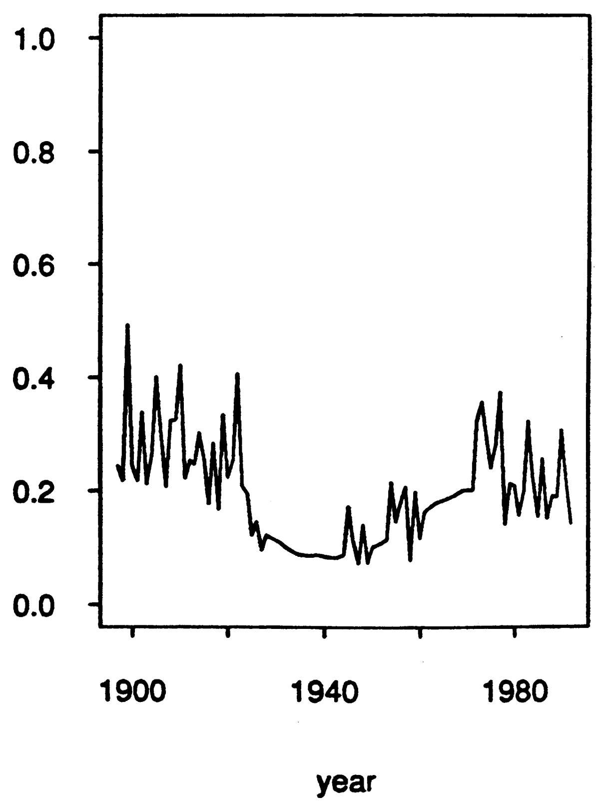


Figure 11: Naive rate Chinese events 1000-1976