Are the 1986-1988 Changes in Solar Free-Oscillation Splitting Significant?

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ABSTRACT

The solar normal mode splitting coefficients for BBSO data differ between 1986 and 1988; inversions for equatorial rotation are slower at depth and faster near the surface in 1988 than in 1986. The significance of the change is disputed. The data sets overlap for 5 splitting coefficients $\{a_j\}_{j=1}^{5}$ for 710 modes. On the assumption that rotation rate varies smoothly with radius, both data sets are satisfied (within the published uncertainties) by the same rotation model at all colatitudes except near 30°- 40° and near 70° (and at their southern hemisphere reflections 140°-150° and 110°). The evidence for equatorial changes is weak. Nonparametric tests show a significant offset in the magnitudes of a_1, a_2 and a_4, and of linear combinations sensitive to rotation at colatitudes of 60°-80° (and 100°-120°). Nonparametric tests show significant radial trends in the changes to a_2, a_4, and (less significantly) a_5. The evidence for radial trends in the changes to rotationally sensitive combinations of a_1, a_3 and a_5 is weak. There is strong anticorrelation between a_2 and a_4, a_1 and a_3, and a_3 and a_5, suggesting the estimates are not independent. Individual coefficients a_j show more evidence for change than do "physical" linear combinations, adding weight to this hypothesis. Some of the changes in splitting might be related to solar activity, which changed most near colatitude 70° from 1986 to 1988.
Subject headings: Sun: rotation—Sun: oscillations—Sun: interior

1 INTRODUCTION

Libbrecht (1989) and Libbrecht and Woodard (1990) provide splitting coefficients $\{a_j^{ln}\}_{j=1}^{s}$, $s = 5$ or 6, characterizing solar free-oscillation spectra for the years 1986 and 1988 obtained from observations at the Big Bear Solar Observatory (BBSO). The splitting coefficients are the coefficients of Legendre polynomial expansions of the shifts of individual singlet frequencies ν_{nlm} (as a function of m/l) from the central frequency ν_{nl0} of the multiplet. Here n is the order of the mode, l is the degree, and m is the azimuthal order. Splitting of a multiplet into singlets with different frequencies that depend on m occurs in theory when the spherical symmetry of the system is broken, for example by rotation or asphericity in the structure of the Sun. Coefficients a_1, a_3, and a_5 depend on rotation: the angular velocity at colatitude θ is linearly related to

$$S_\theta \equiv a_1 + (1 - 5\cos^2\theta)a_3 + (1 - 14\cos^2\theta + 21\cos^4\theta)a_5. \tag{1}$$

(See Brown *et al.* 1989.) This is an even function of $\cos\theta$; north-south asymmetry can not be detected using frequency information alone. Coefficients a_2 and a_4 depend on asphericity of the Sun and are insensitive to angular velocity.

Goode and Dziembowski (1991) and Schou (1991) inverted these two sets of data and found changes in the Sun's internal equatorial rotation rate between 1986 and 1988. Schou claims the differences are not significant, whereas Goode and Dziembowski say they are, and present evidence that the changes are related to the solar cycle. This disagreement has prompted us to take a closer look at the data.

The angular velocity is related to the splitting (approximately) by an integral of the form

$$S_\theta = \int K_\theta \Omega dr, \tag{2}$$

where K_θ is a nonnegative function of the radial coordinate r, and Ω is the angular velocity. Within this approximation, S_θ changes only if the angular velocity distribution changes, although changes in the angular velocity need not change the rotational splitting. Thus we may reduce the question of the necessity of change in angular velocity to the question of change in the splitting coefficients. In particular, we try to answer the questions:

Q1: Can the 1986 and 1988 splitting data be fit adequately by a single model, or are differences in the angular velocity required to account for the splitting?

Q2: Is there a significant change to the "typical" amount of splitting?

Q3: Is there a significant radial trend in the changes?

Q4: Are the odd coefficients determined better or worse than linear combinations of them that are sensitive to rotation?

2 Data, Assumptions and Statistical Caveats

As always, assumptions about the data play a heavy role in the conclusions, and the use of statistical tests invites the abuse of statistical tests. We have tried to minimize the assumptions and abuses.

2.1 Data

The "data" in this study are the splitting coefficients obtained by Libbrecht (1989) and Libbrecht and Woodard (1990) for the BBSO observations. The intersection of the two datasets comprises the first 5 coefficients of 710 modes with frequencies ranging from 1516.1μHz to 3965.5 μHz, and values of l between 5 and 60. For each (l, n) pair, the five coefficients $\{a_j^{ln}\}$ are found by a nonlinear least-squares procedure that fits the splitting coefficients, the background power near the frequency, and the amplitudes of the individual singlets to an estimate of the spatio-temporal power spectrum of the solar surface motions derived from transformed Doppler observations. (See Libbrecht 1989 and Libbrecht and Woodard 1990 for details.)

In the linearization of the problem about a least-squares solution, the covariance matrix of the fitted coefficients is diagonally dominant, since the Legendre polynomials are nearly orthogonal with respect to sums at the sample points (K.G. Libbrecht 1992, personal communication). The diagonal dominance increases as l (and hence the number of sample points in the power spectrum) increases. The errors in the data to which the expansion is fitted are usually taken to be Gaussian by invoking the central limit theorem: the spectra involve linear combinations of vast numbers of the original Doppler observations, which are not totally dependent. The errors in the splitting coefficients are then approximately Gaussian and independent, with standard deviations given by Libbrecht (1989) and Libbrecht and Woodard (1990). (To find a formal covariance matrix would require repeating their analysis, and would give information only locally, in the neighborhood of the least-squares solution. Below we compute empirically the Spearman rank correlation of changes in the estimated coefficients.)

We hope to get an idea of the variability and covariance of splitting coefficients empirically. The distributional details of the errors and uncertainty estimates are not important for the tests we perform, except in section 3.1 where we ask whether the rotation rate must have changed to account for the observed changes in splitting. We try to state clearly the assumptions used in each section.

2.2 Caveats

When performing multiple tests on the same data there is a risk of "data mining:" if we examine enough projections of the data there is a good chance we can find some projections that give apparently but erroneously highly "significant" results. Significance levels should be adjusted to take account of the fact that more than one test is being made. One way to do this and maintain an upper bound on the probability of a type I error (rejecting the null-hypothesis when it is true) is as follows:

Let F be the event that one or more of the N null hypotheses is incorrectly rejected when all the null hypotheses are true (we assume it is possible for all to be true simultaneously), and let F_i be the event that the ith null hypothesis is incorrectly rejected. Let $P(F_i) = p_i$, where $P(\cdot)$ denotes the probability of the event in parentheses. Then

$$P(F) = P(\cup_{i=1}^{N} F_i) \leq \sum_{i=1}^{N} p_i; \tag{3}$$

that is, the chance of at least one type I error is at most the sum of the chances of type I errors in the original tests. We perform dozens of tests and for this reason would like to keep the significance level especially small (< 0.0001 in most of the paper).

Our test for radial trends in the splitting is not strictly legitimate, since the models of Goode and Dziembowski (1991) and Schou (1991), based on the same data, suggested to us such trends might exist. Hypotheses should really be formulated before examining the data used to test them; for example, they might be suggested by a physical theory of how the distribution of angular momentum in the Sun changes over time, or by different data. We await the publication of splitting coefficients for 1990 and 1991 BBSO data to make a better test.

3 The Tests and Their Outcomes

We have performed a number of significance tests on various combinations of the two sets of splitting coefficients. The maximum depth to which a mode is sensitive is monotone in $w \equiv \nu/(l + 1/2)$, where ν is the central frequency of the multiplet. We computed w for each mode and sorted the modes on their w values in order of increasing maximum depth of sensitivity. (For some tests, such as the chi-squared and Wilcoxon signed-rank test, sorting is irrelevant—only the pairing of observations for the same l and n matters.)

We have tested the hypothesis of no change between 1986 and 1988 against several alternatives: any change whatsoever (the omnibus alternative), a shift in the "typical" splitting, and a radial trend in the changes. The omnibus alternative was tested parametrically, relying on the assumed normality of the estimates, their independence, and their published uncertainties. To test for changes in the typical value and for trends we used nonparametric tests based on ranks, without reference to the published uncertainties. Tests against specific alternatives are usually more powerful than tests against the omnibus alternative (they have smaller probability of a type II error, namely, failing to reject the null hypothesis when the alternative is true), and parametric tests tend to be more powerful than nonparametric tests, when the parametric assumptions hold.

3.1 Omnibus Test for Change

Here we examine the evidence for change of any kind in the splitting, and in particular we determine whether the splitting changes require different rotation models. We assume in this section that the published uncertainty estimates are reasonable and that the errors are independent and not too far from normal. We also assume that "the Sun does not look like an onion" (*i.e.* rotation rate does not vary wildly with radius) to justify taking averages of linear combinations of the splitting coefficients for modes with turning points at nearly the same depth (small non-overlapping ranges of w).

In order to determine the change in angular velocity required to account for the observed change in splitting coefficients, we must specify how well (and in what sense) the true splitting should agree with the estimated splitting. Our model for the observational errors is

$$a_i = a_i^0 + \epsilon_i, \tag{4}$$

where a_i is the estimate of the coefficient, a_i^0 is its true value and ϵ_i is random error. According to K.G. Libbrecht (1992, personal communication), the errors ϵ_i may be taken to be independent, zero-mean random variables with standard deviations $\{\sigma_i\}$ given by Libbrecht (1989) and Libbrecht and Woodard (1990). Then the error in a linear combination

$$\lambda = \sum_{i=1}^{I} \alpha_i a_i \tag{5}$$

is

$$\epsilon = \sum_{i=1}^{I} \alpha_i \epsilon_i \tag{6}$$

which has variance

$$\sigma^2 = \sum_{i=1}^{I} \alpha_i^2 \sigma_i^2. \tag{7}$$

Using rules (5)-(7) we construct vectors of differences between linear combinations of the 1986 coefficients and the corresponding 1988 coefficients. That is, we construct a J-vector $\boldsymbol{\delta}$ whose components δ_j are averages (over multiplets with similar turning depths) of differences between values of λ for 1986 and

1988. To test whether the differences are significant, we evaluate the weighted two-norm of the vector of differences $\boldsymbol{\delta}$:

$$||\delta||_\tau \equiv \sqrt{\sum_{j=1}^{J}\left(\frac{\delta_j}{\tau_j}\right)^2}, \tag{8}$$

where τ_j is the standard deviation of δ_j. If $\{\delta_j\}$ are independent zero-mean random variables, the distribution of $z = ||\delta||_\tau^2$ is approximately the chi-squared distribution with J degrees of freedom. For $J > 30$ the distribution of $\sqrt{2}||\epsilon||_\tau - \sqrt{2J-1}$ is approximately the standard normal. By comparing norms of the data with critical values of the normal distribution, we can estimate the probability that the differences would be as large as observed (the p-value), under the null-hypothesis that the values a_i^0 are the same in both years and that the errors ϵ_i are independent zero-mean random variables with standard deviation σ_i and nearly normal distributions. Table 1 gives z-scores and p-values for this hypothesis when different numbers of observations are averaged together in bins in w (the number of degrees of freedom ranges from 710, when there is no averaging, to 71, when sets of 10 observations are averaged; in any case, the approximation of the chi-squared distribution by the normal should be excellent). In the bin averaging, we weighted observations by the inverse of their standard deviations to obtain minimum-variance averages.

The changes in a_1, a_2, a_4, S_{30}, S_{40} and S_{70} appear to be significant, and S_{20} marginally so. The z-scores for the even coefficients are quite high, both with and without averaging, while those of the odd coefficients and linear combinations of them are typically substantially lower when the data are averaged with depth. The decreased significance with small amounts of averaging leads us to doubt the reality of the changes in a_3, a_5, S_0, S_{10}, S_{20}, S_{50}, S_{60} S_{80} and S_{90}, since the significance depends on rapid fluctuations over small depth ranges, which *a priori* we deem unlikely. S_{70} stands out particularly with a strong local maximum of the z-score.

If the average is legitimate, the data are evidence for changes in angular velocity at $30° - 40°$ and at about $70°$; elsewhere no change in the rotation rate is required to fit the data at about the 95% confidence level. The empirical covariance of $\{a_j\}$ (section 3.4) casts some doubt on the significance of the changes at small colatitude. The evidence for change in the equatorial rotation rate is extremely weak, even without averaging. Interestingly, the colatitude of maximum change in sunspot activity from 1986-1988 was near $70°$.

3.2 Test for Shift in the Typical Value

A more specific alternative hypothesis is that the splitting coefficients increased or decreased on the whole over 1986-1988; this alternative can be tested non-parametrically using Wilcoxon's signed rank test (Lehmann 1975). The null-hypothesis for this test is that the coefficients (or linear combinations) are in-

coeff.	z : ave 1	p : ave 1	z : ave 5	p : ave 5	z : ave 10	p : ave 10
a_1	5.67	< 0.0001	5.35	< 0.0001	6.59	< 0.0001
a_2	9.27	< 0.0001	11.64	< 0.0001	12.79	< 0.0001
a_3	3.48	0.0003	1.81	0.0351	2.73	0.0032
a_4	7.45	< 0.0001	7.16	< 0.0001	8.93	< 0.0001
a_5	3.12	0.0009	-1.01	0.8438	-0.40	0.6554
S_0	7.74	< 0.0001	1.48	0.0694	2.16	0.0154
S_{10}	7.99	< 0.0001	1.63	0.0526	2.33	0.0099
S_{20}	8.85	< 0.0001	2.16	0.0154	2.91	0.0018
S_{30}	10.47	< 0.0001	3.34	0.0004	4.25	< 0.0001
S_{40}	6.62	< 0.0001	3.12	0.0009	4.06	< 0.0001
S_{50}	-1.20	0.8849	-1.63	0.9484	-1.01	0.8438
S_{60}	1.54	0.0618	-0.78	0.7823	-0.21	0.5832
S_{70}	3.78	< 0.0001	3.20	0.0006	4.32	< 0.0001
S_{80}	-4.72	> 0.9999	-1.23	0.8907	0.25	0.4013
S_{90}	-4.38	> 0.9999	-2.25	0.9878	-0.94	0.8264

Table 1: z-scores and approximate p-values based on the chi-squared statistic for the hypothesis of no change, when different numbers of observations with neighboring turning points are averaged. Column 1: coefficient or linear combination of coefficients. Columns 2, 4, 6: z-scores for norms of the change in the coefficient or linear combination when 1, 5, or 10 modes (respectively) with neighboring turning points are averaged. Columns 3, 5, 7: p-values corresponding to the z-scores. The weights in the averages are inversely proportional to the uncertainty estimates, yielding averages with minimum variance if the errors are independent. Values for S_θ in the southern hemisphere ($\theta > 90°$) may be obtained by reflection.

coef.	T	z	p
a_1	9.96×10^4	-4.876	< 0.0001
a_2	6.87×10^4	-10.51	< 0.0001
a_3	1.20×10^5	-1.177	0.2391
a_4	1.74×10^5	8.834	< 0.0001
a_5	1.36×10^5	1.844	0.0651
S_0	1.34×10^5	1.370	0.1707
S_{10}	1.33×10^5	1.329	0.1838
S_{20}	1.26×10^5	1.159	0.2465
S_{30}	1.31×10^5	0.8328	0.4050
S_{40}	1.24×10^5	-0.4368	0.6623
S_{50}	1.16×10^5	-1.835	0.0665
S_{60}	1.07×10^5	-3.586	0.0003
S_{70}	9.76×10^4	-5.231	< 0.0001
S_{80}	1.26×10^5	-3.400	0.0007
S_{90}	1.17×10^5	-1.728	0.0841

Table 2: Values of the Wilcoxon signed-rank test statistic T, corresponding z-scores and approximate p-values for the null-hypothesis of no change in the magnitude of the splitting coefficients (or combinations of them) between 1986 and 1988, against the alternative hypothesis of a shift in the typical splitting. The statistic was computed for 1986 minus 1988: negative values of the test statistic indicate the typical value of the coefficient a_i or the combination S_θ of coefficients was larger in 1988 than in 1986.

dependent, and both members of a 1986-1988 pair have the same distribution. The results of the test are in Table 2.

The test shows significant shifts in a_1, a_2, and a_4, and to linear combinations corresponding to rotation changes in the range 60°-80° (most near 70°), and their southern hemispherical reflections. The evidence for shifts in the amount of splitting at other colatitudes is weak, especially near the poles.

3.3 Tests for Radial Trends

A different alternative hypothesis is that there is a radial trend in the changes to the splitting coefficients, which we tested with Spearman's rank correlation coefficient ρ_S (Lehmann 1975)—see Table 3. The null-hypothesis for this test is that the differences between 1986 and 1988 are independent; the alternative hypothesis is that there is an association with radius.

None of the odd coefficients nor linear combinations of them shows a trend at significance level 0.0001. At significance level 0.01, only a_5 and S_0 show

coef.	ρ_S	z	p
a_1	0.064	1.70	0.0890
a_2	−0.259	−6.897	0.0001
a_3	−0.051	−1.363	0.1729
a_4	0.233	6.208	0.0001
a_5	0.106	2.811	0.0049
S_0	0.097	2.579	0.0099
S_{10}	0.096	2.556	0.0106
S_{20}	0.094	2.490	0.0128
S_{30}	0.092	2.457	0.0140
S_{40}	0.065	1.741	0.0817
S_{50}	0.014	0.384	0.7008
S_{60}	−0.063	−1.681	0.0928
S_{70}	−0.043	−1.140	0.2543
S_{80}	0.044	1.171	0.2416
S_{90}	0.080	2.138	0.0325

Table 3: Values of Spearman's rank correlation coefficient ρ_S, corresponding z-scores and approximate p-values for the null-hypothesis of no radial trend in the change in coefficients (or combinations of them) between 1986 and 1988.

significant trends (S_{10} just misses), and S_0 (the poles) only barely so. A 1% probability of random occurence is low, but not overwhelmingly so; it is especially suspect since we are performing so many tests, and because the angular velocity in the polar regions contributes only weakly to the frequency splitting coefficients.

3.4 Correlation among the Coefficients

We also used Spearman's correlation coefficient ρ_S to see whether the coefficients $\{a_j\}_{j=1}^{5}$ are correlated with each other—see Table 4.

The high negative correlation (significance $p < 0.0001$) among even and odd coefficients supports the hypothesis that sums of the odd coefficients are determined better than the odd coefficients individually, and that differences are probably determined worse. The correlation is expected since the least-squares procedure uses only a finite number of points in the power spectrum, the Legendre polynomials are not really orthogonal with respect to the data inner product, leading to tradeoff among even and among odd coefficients. The combinations S_θ involve differences for small colatitudes; all three terms are positive for $\theta \gtrsim 70°$, so we expect angular velocity to be determined better near the equator than near the poles. This is also expected from the observa-

coeff.	ρ_S	z	p
$a_1 - a_2$	0.051	1.368	0.1714
$a_1 - a_3$	−0.234	−6.222	< 0.0001
$a_1 - a_4$	−0.034	−0.918	0.3585
$a_1 - a_5$	−0.064	1.711	0.0871
$a_2 - a_3$	0.037	0.989	0.3225
$a_2 - a_4$	−0.290	−7.727	< 0.0001
$a_2 - a_5$	−0.024	−0.643	0.5205
$a_3 - a_4$	−0.008	−0.207	0.8357
$a_3 - a_5$	−0.295	−7.853	< 0.0001
$a_4 - a_5$	0.095	2.534	0.0113

Table 4: Spearman's rank correlation coefficient ρ_S for changes in pairs of coefficients between 1986 and 1988. There is significant negative correlation between a_1 and a_3, between a_3 and a_5, and between a_2 and a_4, as is expected from the lack of orthogonality of the Legendre polynomials with respect to finite sums.

tion geometry—polar regions are smaller, less well observed and noisier than equatorial regions.

4 Discussion

4.1 Correlation of Equatorial Rotation with Sunspot Number

Figure 2 of Goode and Dziembowski (1991) shows inversions for the equatorial rotation rate at $0.4R$ and the inverse of the mean sunspot number, at five epochs from 1983 to late 1988. Both plots exhibit maxima at the third point (in late 1986), suggesting a relation between rotation and sunspot number. One might like to know the chance of this occurring randomly.

The mean sunspot number is a fixed curve with a minimum at the third point, and increasing monotonically on both sides. Since one might equally well have plotted the mean sunspot number as its inverse, the appropriate question is how likely it is that five random points (representing the rotation rate inversions) have an extremum at the third point, and monotonic behavior either side of the extremum. Inspection of the error bars on the third and fourth points in Figure 2 of Goode and Dziembowski (1991) shows the maximum of the rotation rate may have occurred at the fourth rather than the third point; since the sign of the time axis is not important we are led to seek the probability that five random points have an extremum at the second, third, or fourth point, and monotonic behavior about the extremum.

Were the data independent and identically distributed (iid), all $5! = 120$ possible orderings of their ranks would have equal probability. The probability of an extremum at the second, third or fourth point is the number of orderings giving such an extremum, divided by 120. Enumeration (using symmetries) gives 28 such orderings, so the probability of a coincidental maximum is $7/30 \simeq 0.233$ if the rotational inversions were iid noise. As a result, Goode and Dziembowski's (1991) Figure 2 does not persuade us that the correlation of the inferred rotation near the equatorial plane with the solar-cycle is significant.

4.2 Splitting and Active Regions

The strongest evidence for changes in the splitting we found were at colatitudes near $30° - 40°$ and near $70°$. In particular, the Wilcoxon signed-rank test (Table 2) shows much more significant change near $70°$ than elsewhere. While the observed changes may well reflect changes in solar angular velocity, there are other possibilities. For example, we may be committing a Type II error—incorrectly rejecting the null hypothesis of no change, simply because we observed data that are unlikely, but possible, under the null hypothesis. Other possibilities include the violation of assumptions we made in the analysis, such as independence, validity of the uncertainties assigned by the observers, and perhaps most importantly, the assumption that the splitting coefficients are related to the angular velocity by equations (1) and (2). The increased solar activity near $70°$ in 1988 suggests a different alternative from change in the angular velocity: the estimation of splitting coefficients might be affected by solar activity.

It has been observed recently that the amplitude of acoustic waves is lower in regions of high activity (Brown *et al.* 1992). The spatial distribution of sunspots has energy at azimuthal frequencies comparable to those of the modes we study here. Sunspots superrotate relative to the photosphere, and inversions for rotation in the interior for 1988, using the same data we analyze here, show rotation declining slightly with depth near colatitude $70°$ (Gough *et al.* 1992). Therefore it is plausible that sunspots superrotate relative to the material in the acoustic cavity at colatitude $70°$. The reduced amplitudes of waves near active regions superposes a set of "troughs" that might be precessing faster than the acoustic modes. One would expect this to bias estimates of the splitting upwards. This is borne out by the negative sign of the Wilcoxon statistic for S_{70} in table 2—the splitting was typically larger in 1988 than in 1986.

This potential explanation can not account for the observed changes in linear combinations sensitive to angular velocity near $30° - 40°$.

5 Conclusions

The evidence for changes in the Sun's internal rotation rate between 1986 and 1988 from the BBSO splitting data given by Libbrecht (1989) and Libbrecht and

Woodard (1990) is weak except at colatitudes between 30° and 40° and near 70°. The strongest evidence for change in the typical size of the coefficients is at colatitude 70°, where the data are most likely to have been affected by changes in sunspot activity. The evidence for changes in the equatorial rotation rate is weak.

The coefficients pairs $a_1 - a_3$, $a_2 - a_4$ and $a_3 - a_5$ are significantly correlated. We do not know if this is reflected in the uncertainty estimates given by Libbrecht (1989) and Libbrecht and Woodard (1990).

The answers to questions Q1-Q4 are as follows:

A1: A chi-squared test (Table 1) suggests that the 1986 and 1988 data require different angular velocity models near colatitudes $30° - 40°$ and 70°. Elsewhere, the rotation need not vary with time. Individual splitting coefficients a_1, as well as a_2 and a_4 (unassociated with rotation), changed significantly.

A2: From the Wilcoxon signed-rank test (Table 2), there appears to be a change in the "typical" splitting for coefficient a_1 and the rotation-sensitive linear combination for colatitude 70°. Coefficients a_2 and a_4, which are not associated with rotation, have changed significantly as well.

A3: The Spearman rank correlation test (Table 3) finds strongly significant radial correlations for coefficients a_2 and a_4 (which are not associated with rotation). Some rotation-sensitive linear combinations (S_0 and S_{10}) show evidence of change.

A4: On the chi-squared test, the evidence for change in a_1 is stronger than the evidence for change in coefficients a_3 and a_5, and stronger than the evidence for change in rotationally sensitive linear combinations. On the other hand, the evidence for change in typical values given by the Wilcoxon signed-rank test is stronger for S_{70} than for the odd coefficients a_1, a_3, and a_5. The differences in the smallest significances (most significant changes) are small in both cases. The strong anticorrelation among the odd coefficients found by the Spearman rank correlation test (Table 4) suggests that the reliability of rotation-sensitive linear combinations depends on the latitude (through the signs and magnitudes of the terms). For some latitudes the linear combinations are determined better than the individual coefficients, for other latitudes we believe they are determined worse. A definitive answer to Q4 would require re-analysis of the original spectral data, which is beyond the scope of this work.

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