A Model for Step-stress Accelerated Life Testing Experiments based on Wiener processes and the Inverse Gaussian Distribution.

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1. Introduction

Step-Stress Accelerated Life Testing (SALT) trials are experiments where units of a product or material are run under increasingly severe conditions to get information quickly on its life distribution. The usefulness and analysis of SALT experiments have been discussed by DeGroot and Goel (1979), Nelson (1980), and Bhattacharyya and Soejoeti (1988), among others.

We consider experiments where n units operating independently are put on test at time 0. In the time interval $[0,t]$ the units are subject to stress x_0, while in the interval $[t,\infty)$ the units that have not failed by time t are kept in operation under stress x_1 until they all have failed. Typically, x_0 corresponds to normal stress and x_1 to accelerated stress.

In this paper we consider a fatigue failure model where accumulated fatigue is governed by a Gaussian process that changes at the stress change point t. We show that the distribution of time to failure for this model can be represented in terms of Inverse Gaussian distribution functions, and we show how to estimate the parameters in the model. Extensions to the case of several stress levels are given.

2. A fatigue failure model.

The Inverse Gaussian distribution can be derived as a fatique failure model where accumulated fatigue in time is governed by a Wiener process. See Bhattacharyya and Fries (1982) for a recent discussion.

We develop a model for SALT experiments where the fatigue process changes from one Wiener process to another at the stress change point t. More precisely, in the interval $[0,t]$, we suppose that failure occurs if the process $W_0(y)$ crosses the boundary $\omega > 0$, where $\{W_0(y);\ y \geq 0\}$ is a Wiener process with drift $\eta > 0$ and diffusion constant $\delta^2 > 0$. At the time point t, if $W_0(y)$ has not yet crossed ω in $[0,t]$, the stress is changed from x_0 to x_1 and a new process $W_1(y)$ starts out at the point $(t, W_0(t))$. We assume that

$$W_1(y) = W_0(t + \alpha[y - t]), \quad y > t, \ \alpha > 0$$

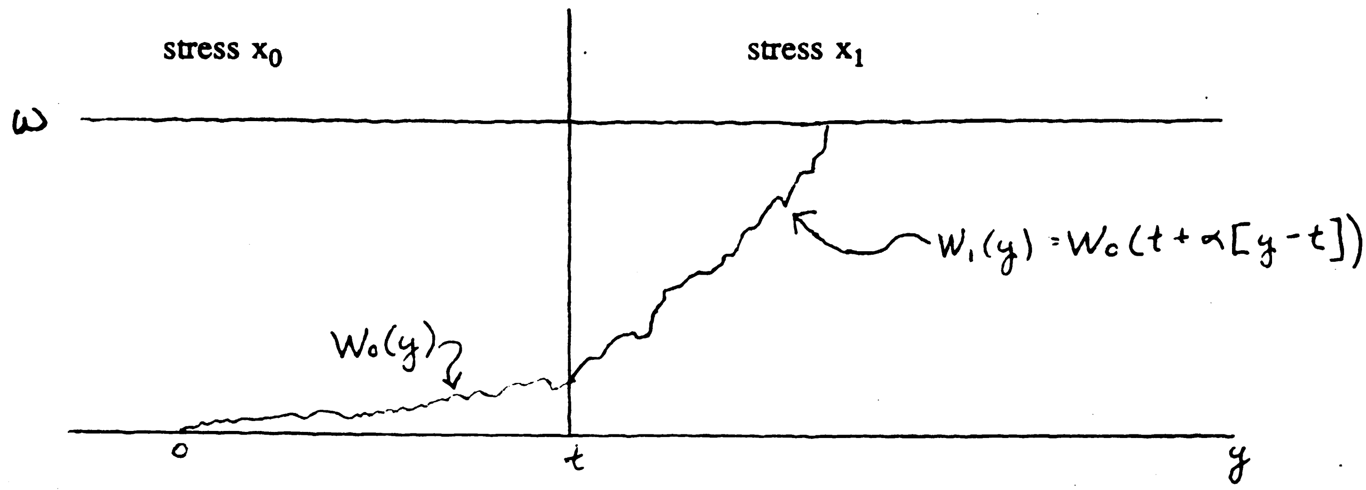

Figure 1.

Our SALT fatigue process is

$$\begin{aligned} W(y) &= W_0(y), & y \leq t \\ &= W_0(t + \alpha[y - t]), & y > t. \end{aligned}$$

Let Y be the first point at which the process W(y) crosses the boundary ω. To derive the distribution of Y, we introduce

$$W^*(y) = \sup_{0 \leq s \leq y} W(s), \quad W_0^*(y) = \sup_{0 \leq s \leq y} W_0(s).$$

Note that the event "$Y > y$" is equivalent to "$W^*(y) < \omega$". For $y \leq t$, this is in turn equivalent to '$W_0^*(y) < \omega$". It is known that $\Pr(W_0^*(y) < \omega)$ is the probability that an Inverse Gaussian random variable with parameters $\mu = \omega/\eta$ and $\lambda = \omega^2/\delta^2$ exceed y. Thus we have

$$F(y) = \Pr(Y \leq y) = IG(y \mid \mu, \lambda), \ y \leq t$$

where $IG(y \mid \mu, \lambda)$ stands for the Inverse Gaussian distribution function with parameters μ and λ. The density of this distribution is

(1) $$f_0(y) = \sqrt{\frac{\lambda}{2\pi y^2}} \exp\{-\frac{\lambda}{2\mu^2} \frac{(y-\mu)^2}{y}\}, \quad y > 0, \ \mu > 0, \ \lambda > 0.$$

Next, for $y > t$, the event "$Y > y$" is again equivalent to "$W^*(y) < \omega$". Note that

$$\text{“}W^*(y) < \omega\text{”} \iff \text{“}\sup_{0\le s\le t} W_0(t) < \omega \text{ and } \sup_{t\le s\le y} W_0(t+\alpha[s-t]) < \omega\text{”}.$$

As s ranges from t to y, $t + \alpha(s - t)$ ranges from t to $t + \alpha(y - t)$. Thus we have

$$\text{“}W^*(y) < \omega\text{”} \iff \text{“}W_0^*(t+\alpha[y-t]) < \omega\text{”}.$$

But it is known that $\Pr(W_0^*(t+\alpha[y-t]) < \omega)$ is the probability that an Inverse Gaussian random variable with parameters μ and λ exceed $t+\alpha[y-t]$. In other words,

$$F(y) = \Pr(Y \le y) = IG(t+\alpha[y-t]\,|\,\mu,\lambda), \quad y > t.$$

To extend the result to $k + 1$ stress levels $x_0, x_1, \ldots, x_k$ over the $k + 1$ intervals $[0, t_1], [t_1, t_2], \ldots, [t_k, \infty)$, we let

$$\begin{aligned} W(y) &= W_0(y), \quad y \in [0, t_1) \\ &= W_i(y), \quad y \in [t_i, t_{i+1}), \; i = 1, \ldots, k, \; t_{k+1} = \infty \end{aligned}$$

where

$$W_i(y) = W_{i-1}(t_i + \alpha_i[y - t_i]), \, y \in [t_i, t_{i+1}), \; i = 1, \ldots, k$$

This expression can be rewritten in terms of W_0 as

$$W_i(y) = W_0(\tau_i + \beta_i[y - t_i]), \; y \in [t_i, t_{i+1}), \; i = 1, \ldots, k$$

where

$$\tau_i = \sum_{j=1}^{i} \beta_{j-1}(t_j - t_{j-1}) \text{ and } \beta_i = \prod_{j=0}^{i} \alpha_j, \; i = 0, \ldots, k, \; \tau_0 = 0$$

where $t_0 = 0$ and $\alpha_0 = 1$. Using the preceding arguments, we find that the time Y to failure of one unit has distribution function

$$F(y) = IG(\tau_i + \beta_i[y - t_i]\,|\,\mu,\lambda), \quad y \in [t_i t_{i+1}), \quad i = 0, \ldots, k.$$

3. Estimation in the Inverse Gaussian SALT model.

In the previous section we derived a model where the distribution of the survival time Y for one unit in a two stress level SALT experiment is

$$\begin{aligned} F(y) &= F_0(y), \quad 0 \le y \le t \\ &= F_0(t + \alpha[y - t]), \; y > t \end{aligned} \tag{2}$$

Here F_0 is the survival distribution under the normal stress x_0. We also found that for our Gaussian process model F_0 is the Inverse Gaussian distribution whose density is given by (1).

Models of the type (2) have been considered by De Groot and Goel (1979) and Nelson (1980) who used exponential and Weibull distributions for F_0 rather than the Inverse Gaussian distribution.

In our Inverse Gaussian SALT model, given by (1) and (2), the mean survival time μ and survival distribution F_0 under the normal stress x_0 is of particular interest. We next show how to obtain estimates of μ, λ and α, and thus also F_0, from the outcomes of SALT experiments.

Consider the likelihood corresponding to models (1) and (2). For an observed sample $y_1, \ldots, y_n$ of failure times, let $y_{(1)} < y_{(2)} < \cdots < y_{(n)}$ denote the ordered failure times and let r be the number of failures at or before time t so that

$$y_{(r)} \leq t < y_{(r+1)}.$$

For the time being, we consider $1 \leq r \leq n-1$. Then the likelihood corresponding to model (2) can be written

$$\begin{aligned} L(\mu, \lambda, \alpha) &= \prod_{i=1}^{n} \{[f_0(y_i)]^{I[y_i \leq t]} [f_0(y_i + \alpha(y_i - t))]^{I[y_i > t[} \\ &= \{\prod_{i=1}^{r} f_0(y_{(i)})\} \{\prod_{j=r+1}^{n} f_0(y_{(j)} + \alpha(y_{(j)} - t))\} \end{aligned}$$

where $I[A]$ denotes the indicator of the event A. When f_0 is the IG density (1), the log likelihood $l(\mu, \lambda, \alpha) = \log L(\mu, \lambda, \alpha)$ is

$$\begin{aligned} l(\mu, \lambda, \alpha) &= n\left(\frac{\lambda}{\mu} + \frac{1}{2}\log\frac{\lambda}{2\pi}\right) - \frac{3}{2}\sum_{i=1}^{r} \log y_{(i)} - \frac{\lambda}{2\mu^2}\sum_{i=1}^{r} y_{(i)} - \frac{1}{2}\lambda\sum_{i=1}^{r} y_{(i)}^{-1} \\ &- \frac{3}{2}\sum_{j=r+1}^{n} \log y_{(j)}(\alpha) - \frac{\lambda}{2\mu^2}\sum_{j=r+1}^{n} y_{(j)}(\alpha) - \frac{1}{2}\lambda\sum_{j=r+1}^{n} y_{(j)}^{-1}(\alpha) \end{aligned}$$

where

$$y_{(j)}(\alpha) = t + \alpha(y_{(j)} - t).$$

For fixed α, the likelihood equations can be solved explicitly for μ and λ. In fact, we find

$$\begin{aligned} \hat{\mu}_\alpha &= \frac{1}{n}\left[\sum_{i=1}^{r} y_{(i)} + \sum_{j=r+1}^{n} y_{(j)}(\alpha)\right] = \bar{y} + \frac{(\alpha-1)}{n}\sum_{j=r+1}^{n}[y_{(j)} - t] \\ \hat{\lambda}_\alpha &= \left\{\frac{1}{n}\left[\sum_{i=1}^{r} y_{(i)}^{-1} + \sum_{j=r+1}^{n} y_{(j)}^{-1}(\alpha)\right] - \frac{1}{\hat{\mu}_\alpha}\right\}^{-1} \end{aligned} \tag{3}$$

Next, to compute the maximum likelihood estimate $\hat{\alpha}$ of α, we find the value $\hat{\alpha}$ that maximizes the profile log-likelihood

$$l(\alpha) = \max_{\mu,\lambda} l(\mu,\lambda,\alpha) = l(\hat{\mu}_\alpha, \hat{\lambda}_\alpha, \alpha)$$

with respect to α. Finally, our estimates are $\hat{\alpha}$, $\hat{\mu} = \hat{\mu}_{\hat{\alpha}}$ and $\hat{\lambda} = \hat{\lambda}_{\hat{\alpha}}$.

The three estimates $\hat{\alpha}$, $\hat{\mu}$, $\hat{\lambda}$ are jointly asymptotically tri-variate normal with asymptotic covariance matrix given by the inverse of the Fisher information matrix. Asymptotic confidence intervals can be obtained by estimating the asymptotic covariance matrix by the inverse of the observed Fisher information matrix

$$\frac{1}{n}\frac{\partial}{\partial\theta_i}\frac{\partial}{\theta_j} l(\theta_1,\theta_2,\theta_3), \quad i,j = 1,2,3$$

where $(\theta_1,\theta_2,\theta_3) = (\mu,\lambda,\alpha)$.

Until now we have assumed that $0 < r < n$. If $r = 0$, i.e. there are no failures under normal stress, then α, μ and λ cannot be estimated. In the opposite case where $r = n$, α can not be estimated, but the parameters that count, μ, λ and F_0, can be estimated. In fact, $\hat{\mu}$ and $\hat{\lambda}$ are given by (3) with $r = n$. SALT experiments should be designed so that it is improbable that r is close to zero.

To obtain the maximum likelihood estimate of the survival distribution under the normal stress x_0 we substitute $\hat{\mu}$ and $\hat{\lambda}$ into the formula for $F_0(y)$; thus

$$\hat{F}_0(y) = \Phi\left(\frac{\sqrt{\hat{\lambda}}}{\hat{\mu}}\sqrt{y} - \sqrt{\hat{\lambda}}\frac{1}{\sqrt{y}}\right) + \Phi\left(-\frac{\sqrt{\hat{\lambda}}}{\hat{\mu}}\sqrt{y} - \sqrt{\hat{\lambda}}\frac{1}{\sqrt{y}}\right)e^{2\frac{\hat{\lambda}}{\hat{\mu}}}$$

In $k + 1$ stress level SALT experiments where

$$F(y) = F_0(\tau_i + \beta_i[y - t_i]), \quad y \in [t_i, t_{i+1}), \; i = 0,\dots,k$$

we can extend the above methods to jointly estimate the parameters μ, λ, α_1, $\alpha_2, \dots, \alpha_k$. Alternatively, we can introduce a regression structure

$$\alpha_i = \beta q(x_i), \quad \beta > 0, \; i = 1,\dots,k$$

where q is a known function such as $q(x) = x$ or $q(x) = \sqrt{x}$. Now we can estimate μ, λ and β by maximum likelihood methods. Another possible regression structure would be

$$\alpha_i = \exp\{\beta q(x_i)\}.$$

Alternatively, we could introduce regression structures in terms of the mean μ_i of $F_0(\tau_i + \beta_i[y - t_i])$; i.e. in terms of $\mu_i = t_i + \beta_i^{-1}(\mu - \tau_i)$. Two possible expressions are

$$\mu_i = \beta q(x_i), \quad \text{and}$$

$$\mu_i = \exp\{\beta q(x_i).$$

Which regression structure is most appropriate will have to be determined using model diagnostics and residual analysis.

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